Studio della funzione

$f(x) = x[(-logx)^2 + 2logx]$

$\Leftrightarrow f(x) = x(\log^2 x + 2logx)$

1) Determinare l'insieme di definizione della funzione

   ID: $x > 0;$

   $X =]0, +\infty[$

2) Studio del segno della funzione

   $f(x) = 0; \quad \leftrightarrow \quad x(\log^2 x + 2logx) = 0;$

$$\Leftrightarrow \begin{cases} x = 0\,; \\ oppure \\ \log^2 x + 2logx = 0; \end{cases} \Leftrightarrow \begin{cases} x = 0 \text{ la funzione non è definita;} \\ oppure \\ logx(logx + 2) = 0; \end{cases} \Leftrightarrow \begin{cases} logx = 0; \\ oppure \\ logx + 2 = 0; \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 1; \\ oppure \\ x = e^{-2}; \end{cases}$$

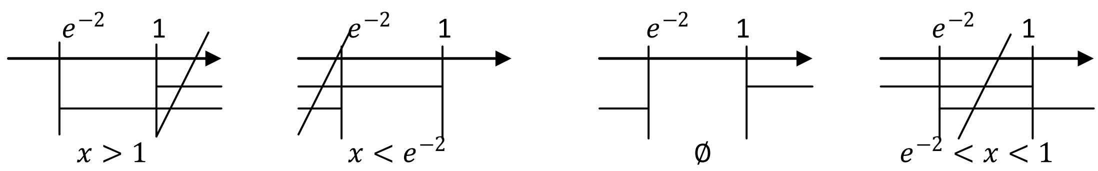
$f(x) > 0; \quad \Leftrightarrow \quad x(\log^2 x + 2logx) > 0\,;$

$$\Leftrightarrow \begin{cases} x > 0; \\ \log^2 x + 2logx > 0; \end{cases} \quad o \quad \begin{cases} x < 0; \\ \log^2 x + 2logx < 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x > 0; \\ logx(logx + 2) > 0 \end{cases} \quad o \quad \begin{cases} x < 0; \\ logx(logx + 2) < 0 \end{cases}$$

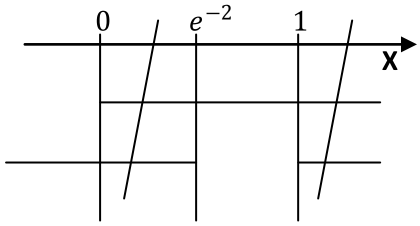
$$\Leftrightarrow \begin{cases} x > 0; \\ \begin{cases} logx > 0; \\ logx + 2 > 0; \end{cases} o \begin{cases} logx < 0; \\ logx < 0; \end{cases} \end{cases} o \begin{cases} x < 0; \\ \begin{cases} logx > 0; \\ logx + 2 < 0; \end{cases} o \begin{cases} logx < 0; \\ logx + 2 > 0; \end{cases} \end{cases}$$

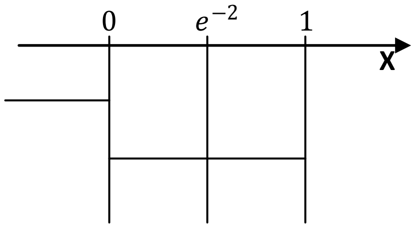
$$\Leftrightarrow \begin{cases} x > 0; \\ \begin{cases} x > 1; \\ x > e^{-2}; \end{cases} o \begin{cases} x < 1; \\ x < e^{-2}; \end{cases} \end{cases} o \begin{cases} x < 0; \\ \begin{cases} x > 1; \\ x < e^{-2}; \end{cases} o \begin{cases} x < 1; \\ x > e^{-2}; \end{cases} \end{cases}$$



$$x < e^{-2};\ \ x > 1; \qquad\qquad e^{-2} < x < 1$$

$$\Leftrightarrow \begin{cases} x > 0; \\ x < e^{-2};\ \ x > 1; \end{cases} \qquad o \qquad \begin{cases} x < 0; \\ e^{-2} < x < 1 \end{cases}$$





$0 < x < e^{-2} \quad \cup \quad x > 1$ $\qquad\qquad \emptyset$

Soluzione:

$f(x) > 0; \qquad 0 < x < e^{-2} \quad \cup \quad x > 1$

$f(x) < 0; \qquad x < 0 \quad \cup \quad e^{-2} < x < 1$

3) Comportamento ai limiti ($\pm\infty$) e ai limiti degli estremi, determinazione degli asintoti della funzione

$$\lim_{x \to +\infty} x(\log^2 x + 2logx) = +\infty$$

$$\lim_{x \to 0} x(\log^2 x + 2logx) = 0$$

4) Studio della derivata prima della funzione, $f'(x) = 0$, $f'(x) > 0$, $f'(x) < 0$, ricerca dei punti di massimo e di minimo, studio l'andamento della funzione.

$f'(x) = D(x)(\log^2 x + 2logx) + xD(\log^2 x + 2logx)$

$$\Leftrightarrow \log^2 x + 2logx + x\left(2logx \cdot \frac{1}{x} + \frac{2}{x}\right)$$

$$\Leftrightarrow \log^2 x + 2logx + 2logx + 2$$

$$\Leftrightarrow \underline{\log^2 x + 4logx + 2}$$

$f'(x) = 0 \ \Leftrightarrow \ \log^2 x + 4logx + 2 = 0$

$$Pongo\ logx = y$$

$$\Leftrightarrow\ y^2 + 4y + 2 = 0$$

$$\Leftrightarrow \qquad y_1 = -2 - \sqrt{2} \qquad , \qquad y_2 = -2 + \sqrt{2}$$

$$\Leftrightarrow \begin{cases} logx = -2 - \sqrt{2}; \\ oppure \\ logx = -2 + \sqrt{2}; \end{cases} \qquad \Leftrightarrow \begin{cases} x = e^{-2-\sqrt{2}} \\ oppure \\ x = e^{-2+\sqrt{2}} \end{cases}$$

$f'(x) > 0 \quad \Leftrightarrow \quad \log^2 x + 4logx + 2 > 0$

$Pongo\ logx = y$

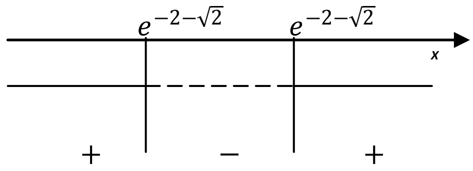
$\Leftrightarrow \ y^2 + 4y + 2 > 0$

$y = -2 \pm \sqrt{4-2} \qquad y_1 = -2-\sqrt{2}; \qquad y_2 = -2+\sqrt{2};$

$y < -2-\sqrt{2} \quad \cup \quad y > -2+\sqrt{2}$

$\Leftrightarrow logx < -2-\sqrt{2} \quad \cup \quad logx > -2+\sqrt{2}$

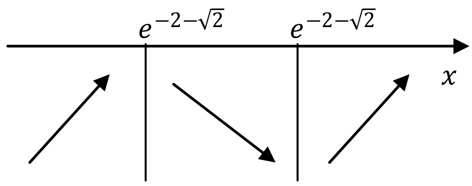
$\Leftrightarrow x < e^{-2-\sqrt{2}} \quad \cup \quad x > e^{-2+\sqrt{2}}$



Soluzione:

$f'(x) > 0 \qquad x < e^{-2-\sqrt{2}} \quad \cup \quad x > e^{-2+\sqrt{2}}$

$f'(x) < 0 \qquad e^{-2-\sqrt{2}} < x < e^{-2+\sqrt{2}}$



$M\left(e^{-2-\sqrt{2}},\ f\left(e^{-2-\sqrt{2}}\right)\right) = \left(e^{-2-\sqrt{2}}, +0{,}16\right)$

$m\left(e^{-2+\sqrt{2}}, f\left(e^{-2+\sqrt{2}}\right)\right) = \left(e^{-2+\sqrt{2}}, -0{,}46\right)$

5) Studio della derivata seconda della funzione, $f''(x) = 0,\ f''(x) > 0,\ f''(x) < 0,$ ricerca dei punti di flesso, studio della concavità e della convessità della funzione

$f''(x) = D\log^2 x + 4logx + 2$

$\Leftrightarrow \ 2logx \cdot \frac{1}{x} + \frac{4}{x}$

$\Leftrightarrow \ \frac{2}{x}(logx + 2)$

$f''(x) = 0 \quad \Leftrightarrow \quad \frac{2}{x}(logx + 2) = 0$

$\Leftrightarrow \quad logx + 2 = 0$

$\Leftrightarrow \quad logx = -2$

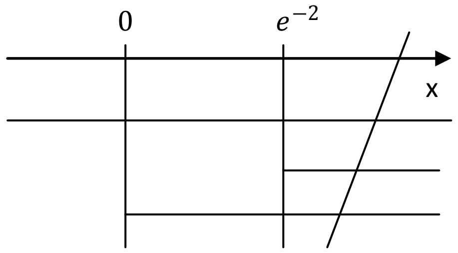
$\Leftrightarrow \quad x = e^{-2}$

$F\left(e^{-2}, f(e^{-2})\right) = (e^{-2}, 0)$

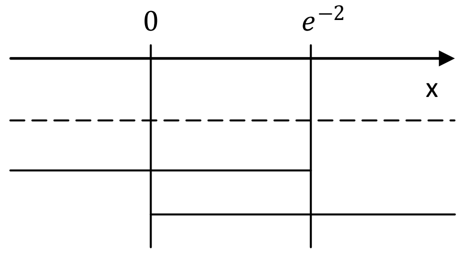
$f''(x) > 0 \quad \Leftrightarrow \quad \frac{2}{x}(logx + 2) > 0$

$\Leftrightarrow \begin{cases} 2 > 0; \\ logx + 2 > 0; \\ x > 0; \end{cases} \qquad o \qquad \begin{cases} 2 < 0; \\ logx + 2 < 0; \\ x > 0; \end{cases}$

$$\Leftrightarrow \begin{cases} \forall x \in \mathbb{R}; \\ x > e^{-2} \\ x > 0; \end{cases} \qquad o \qquad \begin{cases} \emptyset; \\ x < e^{-2}; \\ x > 0; \end{cases}$$



$x > e^{-2}$

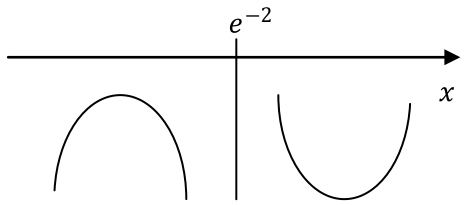


$\emptyset$

Soluzione:

$f''(x) > 0 \qquad x > e^{-2}$

$f''(x) < 0 \qquad x < e^{-2}$



6) Grafico