Individuare l'insieme di definizione delle seguenti disequazioni trigonometriche:
1)

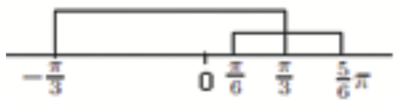
$$f(x) = \frac{(2\sin x - 1)(3^x + 2)}{(x^2+1)(2\cos x - 1)} \qquad \text{da valutare nell'intervallo } \left]-\frac{\pi}{2}, \frac{3}{2}\pi\right[$$

Essendo (3^x+2) e (x^2+1) sempre positive, le soluzioni sono date dal sistema

$$\begin{cases} 2\sin x - 1 > 0 \\ 2\cos x - 1 > 0 \end{cases} \quad \cup \quad \begin{cases} 2\sin x - 1 < 0 \\ 2\cos x - 1 < 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} \sin x > \frac{1}{2} \\ cosx > \frac{1}{2} \end{cases} \quad \cup \quad \begin{cases} \sin x < \frac{1}{2} \\ \cos x < \frac{1}{2} \end{cases} \Leftrightarrow$$

$$\begin{cases} \frac{\pi}{6} < x < \frac{5}{6}\pi \\ -\frac{\pi}{3} < x < \frac{\pi}{3} \end{cases} \quad \cup \quad \begin{cases} -\frac{\pi}{2} < x < \frac{\pi}{6} \quad \cup \quad \frac{5}{6}\pi < x < \frac{3}{2}\pi \\ -\frac{\pi}{2} < x < -\frac{\pi}{3} \quad \cup \quad \frac{\pi}{3} < x < \frac{3}{2}\pi \end{cases}$$



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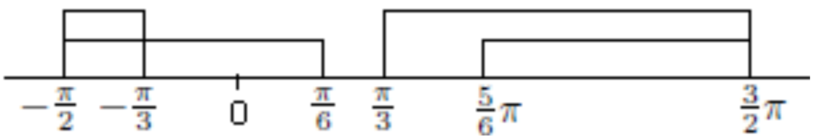
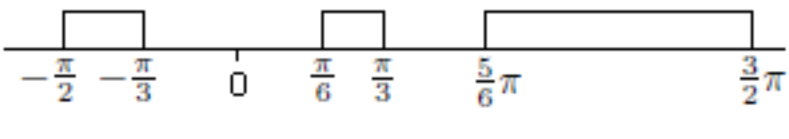


$$x \in \quad \left]-\frac{\pi}{2}, -\frac{\pi}{3}\right[\cup \left]\frac{\pi}{6}, \frac{\pi}{3}\right[\cup \left]\frac{5}{6}\pi, \frac{3}{2}\pi\right[$$



2)

$$f(x) = \frac{(\arccos \frac{2x}{\pi} + 3)(1 - \mathrm{tg}x)}{2\sin x - 1} > 0 \qquad \text{da valutare nell'intervallo } \left] -\frac{\pi}{2}, \frac{3}{2}\pi \right[$$

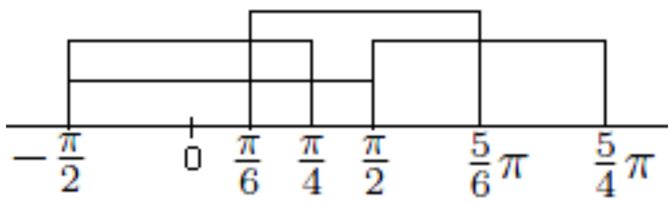
$$f(x) > 0 \Leftrightarrow \begin{cases} (\arccos \frac{2x}{\pi} + 3)(1 - \mathrm{tg}x) > 0 \\ 2\sin x - 1 > 0 \end{cases} \cup \begin{cases} (\arccos \frac{2x}{\pi} + 3)(1 - \mathrm{tg}x) < 0 \\ 2\sin x - 1 < 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} \begin{cases} \arccos \frac{2x}{\pi} + 3 > 0 \\ 1 - \mathrm{tg}x > 0 \end{cases} \cup \begin{cases} \arccos \frac{2x}{\pi} + 3 < 0 \\ 1 - \mathrm{tg}x < 0 \end{cases} \\ \\ 2\sin x - 1 > 0 \end{cases} \cup \begin{cases} \begin{cases} \arccos \frac{2x}{\pi} + 3 < 0 \\ 1 - \mathrm{tg}x > 0 \end{cases} \cup \begin{cases} \arccos \frac{2x}{\pi} + 3 > 0 \\ 1 - \mathrm{tg}x < 0 \end{cases} \\ \\ 2\sin x - 1 < 0 \end{cases}$$

Dato che $\arccos \frac{2x}{\pi} + 3$ è sempre positivo, possiamo riscrivere il sistema come segue:

$$\begin{cases} \begin{cases} \arccos \frac{2x}{\pi} + 3 > 0 \\ 1 - \mathrm{tg}x > 0 \end{cases} \\ \\ 2\sin x - 1 > 0 \end{cases} \cup \begin{cases} \begin{cases} \arccos \frac{2x}{\pi} + 3 > 0 \\ 1 - \mathrm{tg}x < 0 \end{cases} \\ \\ 2\sin x - 1 < 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} \begin{cases} -1 \le \frac{2x}{\pi} \le 1 \\ \frac{\pi}{4} < x < \frac{\pi}{2} \cup \frac{\pi}{2} < x < \frac{5}{4}\pi \end{cases} \\ \\ \frac{\pi}{6} < x < \frac{5}{6}\pi \end{cases} \cup \begin{cases} \begin{cases} -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \frac{\pi}{4} < x < \frac{\pi}{2} \cup \frac{5}{4}\pi < x < \frac{3}{2}\pi \end{cases} \\ \\ -\frac{\pi}{2} < x < \frac{\pi}{6} \cup \frac{5}{6}\pi < x < \frac{3}{2}\pi \end{cases}$$



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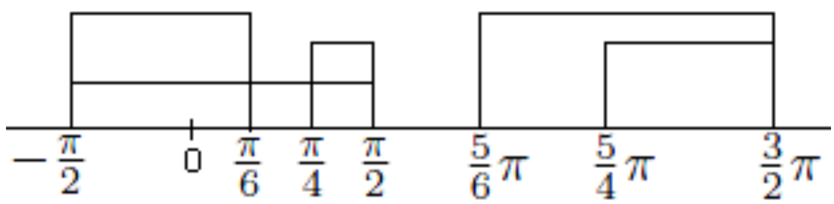
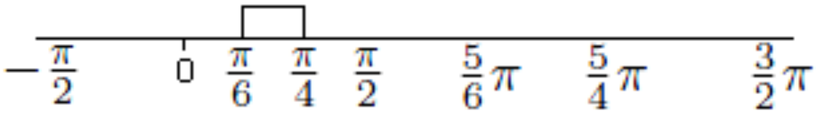


$$x \in \left]\frac{\pi}{6}, \frac{\pi}{4}\right[\quad \cup \quad \varnothing$$



$$x \in \left]\frac{\pi}{6}, \frac{\pi}{4}\right[$$