Esercizi sui numeri complessi

Scrivere in forma algebrica z= a + ib con $a, b \in \mathbb{R}$ i seguenti numeri complessi:
1)

$$\frac{1}{i(3+2i)^2}$$

$$= \frac{1}{i(9+4i^2+12i)} = \frac{1}{i(9+12i-4)} = \frac{1}{9i+12i^2-4i} =$$

$$= -\frac{1}{12-5i} = \frac{1(12+5i)}{(12-5i)(12+5i)} = -\frac{-12-5i}{144+25} = -\frac{12}{169} - \frac{5}{169}i.$$

In questo esercizio, così come nei successivi, moltiplichiamo numeratore e denominatore per il coniugato del denominatore svolgendo poi alcuni passaggi algebrici. Ricordare che dato un numero complesso $z = a + ib$ il suo coniugato $\overline{z}$ è $a - ib$. Notare inoltre che $i^2 = -1$.

2)

$$\frac{(2+i)(1-i)}{3-2i}$$

$$= \frac{(2+i)(1-i)(3+2i)}{(3-2i)(3+2i)} = \frac{(2-2i+i-i^2)(3+2i)}{9-4i^2} =$$

$$= \frac{9+6i-6i-4i^2+3i+2i^2}{9+4} = \frac{11+3i}{13} = \frac{11}{13} + \frac{3}{13}i.$$

3)

$$\frac{(\sqrt{3}+\sqrt{2}i)^3}{\sqrt{2}-\sqrt{3}i}$$

$$= \frac{(\sqrt{3}+\sqrt{2}i)^3(\sqrt{2}+\sqrt{3}i)}{(\sqrt{2}-\sqrt{3}i)(\sqrt{2}+\sqrt{3}i)} = \frac{(3\sqrt{3}+9\sqrt{2}i-6\sqrt{3}-2\sqrt{2}i)(\sqrt{2}+\sqrt{3}i)}{2-3i^2} =$$

$$= \frac{(-3\sqrt{3}+7\sqrt{2}i)(\sqrt{2}+\sqrt{3}i)}{2+3} = \frac{-3\sqrt{6}-9i+14i-7\sqrt{6}}{5} = \frac{-10\sqrt{6}+5i}{5} = -2\sqrt{6}+i.$$

Ridurre in forma trigonometrica i seguenti numeri complessi:
1)

$$z = -3i$$

$$a = 0, b = -3 \rightarrow \varrho = \sqrt{0+9} = 3 \rightarrow \left.\begin{array}{l} \cos\theta = \frac{0}{3} = 0 \\ \\ \sin\theta = \frac{-3}{3} = -1 \end{array}\right\} \rightarrow \theta = \frac{3}{2}\pi$$

$$z = 3(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi).$$

Si ricorda che

$$z = a + ib: \qquad \varrho = \sqrt{a^2 + b^2}, \quad \cos\theta = \frac{a}{\varrho}, \quad sin\theta = \frac{b}{\varrho}, \quad z = \varrho(\cos\theta + i\sin\theta)$$

2)

$$z = -5$$

$$a = -5, b = 0 \to \varrho = \sqrt{25 + 0} = 5 \to \left.\begin{array}{l} \cos\theta = \frac{-5}{5} = -1 \\ \\ \sin\theta = \frac{0}{5} = 0 \end{array}\right\} \to \theta = \pi$$

$$z = 5(\cos\pi + i\sin\pi).$$

3)

$$z = \sqrt{3} + 1$$

$$a = \sqrt{3}, b = 1 \to \varrho = \sqrt{3 + 1} = 2 \to \left.\begin{array}{l} \cos\theta = \frac{\sqrt{3}}{2} \\ \\ \sin\theta = \frac{1}{2} \end{array}\right\} \to \theta = \frac{5}{6}\pi$$

$$z = 2(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi).$$

I seguenti numeri complessi non possono essere direttamente trasformati in forma trigonometrica. Moltiplichiamo quindi numeratore e denominatore per il coniugato del denominatore ed eseguiamo alcuni passaggi algebrici per ridurli alla forma $z = a + ib$.

1)

$$\frac{1+i}{1-i}$$

$$= \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1 + i^2 + 2i}{2} = \frac{2i}{2} = i \text{ quindi } a = 0, b = 1$$

$$\varrho = \sqrt{0 + 1} = 1 \to \left.\begin{array}{l} \cos\theta = \frac{0}{1} = 0 \\ \\ \sin\theta = \frac{1}{1} = 1 \end{array}\right\} \to \theta = \frac{\pi}{2}$$

$$z = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}.$$

2)

$$\frac{1+i}{\sqrt{3}+i}$$

$$= \frac{(1+i)(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)} = \frac{\sqrt{3}+1}{4} + \frac{i(\sqrt{3}-1)}{4}$$

$$\text{quindi } a = \frac{\sqrt{3}+1}{4}, b = \frac{(\sqrt{3}-1)}{4}$$

$$\varrho = \sqrt{\frac{3+1+2\sqrt{3}}{16} + \frac{3+1-2\sqrt{3}}{16}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\left.\begin{aligned} \cos\theta &= \tfrac{\sqrt{3}+1}{4} \cdot \sqrt{2} = \tfrac{\sqrt{6}+\sqrt{2}}{4} \\ \sin\theta &= \tfrac{\sqrt{3}-1}{4} \cdot \sqrt{2} = \tfrac{\sqrt{6}-\sqrt{2}}{4} \end{aligned}\right\} \to \theta = \frac{\pi}{12}$$

$$\text{Notare che } \cos\frac{\pi}{12} = \cos(\frac{\pi}{4} - \frac{\pi}{6}) = \cos\frac{\pi}{4} \cdot \sin\frac{\pi}{6} + \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{6} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$z = \frac{1}{\sqrt{2}}(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}).$$

3)

$$\frac{1+i\sqrt{3}}{1-i}$$

$$= \frac{(1+i\sqrt{3})(1+i)}{(1-i)(1+i)} = \frac{(1+i\sqrt{3})(1+i)}{1-i^2} = \frac{1+i+\sqrt{3}i+\sqrt{3}i^2}{2} = \frac{1-\sqrt{3}}{2} + \frac{i(1+\sqrt{3})}{2}$$

$$\text{Quindi } a = \frac{1-\sqrt{3}}{2}, b = \frac{1+\sqrt{3}}{2}$$

$$\varrho = \sqrt{\frac{1+3-2\sqrt{3}}{4} + \frac{3+1+2\sqrt{3}}{4}} = \sqrt{2}$$

$$\left.\begin{aligned} \cos\theta &= \tfrac{1-\sqrt{3}}{2} \cdot \tfrac{1}{\sqrt{2}} = \tfrac{1-\sqrt{3}}{2\sqrt{2}} \\ \sin\theta &= \tfrac{1+\sqrt{3}}{2} \cdot \tfrac{1}{\sqrt{2}} = \tfrac{1+\sqrt{3}}{2\sqrt{2}} \end{aligned}\right\} \to \theta = \frac{7}{12}\pi$$

$$z = \sqrt{2}(\cos\frac{7}{12}\pi + i\sin\frac{7}{12}\pi)$$

Utilizzare la formula di De Moivre ($z^n = \varrho^n(\cos(n\theta) + i\sin(n\theta))$ per calcolare le potenze dei seguenti numeri complessi :

1)

$$(\frac{1-i}{1+i})^3$$

$$= (\frac{(1-i)^2}{1-i^2})^3 = (\frac{1+i^2-2i}{2})^3 = (\frac{-2i}{2})^3 = -i^3$$

Quindi $a = 0, b = -1$

$$\varrho = \sqrt{0+1} = 1 \qquad \left.\begin{array}{l} \cos\theta = \frac{0}{1} = 0 \\ \\ \sin\theta = \frac{-1}{1} = -1 \end{array}\right\} \rightarrow \theta = \frac{3}{2}\pi$$

$$(\frac{1-i}{1+i})^3 = 1^3(\cos(3 \cdot \frac{3}{2}\pi) + \sin(3 \cdot \frac{3}{2}\pi)i) = \cos\frac{9}{2}\pi + \sin\frac{9}{2}\pi i = 0 + 1i = i$$

2)

$$(1+i)^{20}$$

$$a = 1, \quad b = 1 \qquad \varrho = \sqrt{2}$$

$$\left.\begin{array}{l} \cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \\ \sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{array}\right\} \rightarrow \theta = \frac{3}{2}\pi$$

$$(1+i)^{20} = (\sqrt{2})^{20}(\cos(20 \cdot \frac{\pi}{4}\pi) + \sin(20 \cdot \frac{\pi}{4})i) = 2^{10}(-1+0i) = -2^{10}$$

3)

$$(1-i)^{11}$$

$$a = 1, \quad b = -1 \qquad \varrho = \sqrt{2}$$

$$\left.\begin{array}{l} \cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \\ \sin\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{array}\right\} \rightarrow \theta = \frac{\pi}{4}$$

$$(1-i)^{11} = (\sqrt{2})^{11}(\cos(11 \cdot \frac{7}{4}\pi) + \sin(11 \cdot \frac{7}{4}\pi)i) = 2^5\sqrt{2}(\cos\frac{5}{4}\pi - \sin 54\pi i) = 32\sqrt{2}(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) = -32-32i$$

Utilizzare la formula $z_k = \varrho^{\frac{1}{n}}[\cos(\frac{\theta+2k\pi}{n}) + i\sin(\frac{\theta+2k\pi}{n})]$ per trovare le radici dei seguenti numeri complessi:

1)

$$\sqrt[3]{i-1}$$

$$a=-1, \quad b=1, \qquad \varrho=\sqrt{2} \qquad \left.\begin{array}{l}\cos\theta=\frac{-1}{\sqrt{2}}\\ \\ \sin\theta=\frac{1}{\sqrt{2}}\end{array}\right\} \rightarrow \theta=\pi-\frac{\pi}{4}=\frac{3}{4}\pi$$

$$z_k=\sqrt{2}^{\frac{1}{3}}[\cos(\frac{\frac{3}{4}\pi+2k\pi}{3})+i\sin(\frac{\frac{3}{4}\pi+2k\pi}{3})] \text{ per k = 0, 1, 2}$$

$$z_0=\sqrt{2}^{\frac{1}{3}}[\cos(\frac{\frac{3}{4}\pi}{3})+i\sin(\frac{\frac{3}{4}\pi}{3})]=\sqrt[6]{2}[\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}]$$

$$z_1=\sqrt[6]{2}[\cos(\frac{\frac{3}{4}\pi+2\pi}{3})+i\sin(\frac{\frac{3}{4}\pi+2\pi}{3})]=\sqrt[6]{2}[\cos(\frac{\frac{11}{4}\pi}{3})+i\sin(\frac{\frac{11}{4}\pi+2\pi}{3})]=$$

$$=\sqrt[6]{2}[cos(\frac{11}{12}\pi)+i\sin(\frac{11}{12}\pi)]=\sqrt[6]{2}[-\frac{\sqrt{6}+\sqrt{2}}{4}+i\frac{\sqrt{6}-\sqrt{2}}{4}]$$

$$\text{Essendo } \cos\frac{11}{12}\pi=\cos(\pi-\frac{\pi}{12})=-\cos\frac{\pi}{12}=-\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\text{e } \sin\frac{11}{12}\pi=\sin(\pi-\frac{\pi}{12})=\sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

$$z_2=\sqrt[6]{2}[\cos(\frac{\frac{3}{4}\pi+4\pi}{3})+i\sin(\frac{\frac{3}{4}\pi+4\pi}{3})]=\sqrt[6]{2}[cos(\frac{19}{12}\pi)+i\sin(\frac{19}{12}\pi)]=$$

$$=\sqrt[6]{2}[\frac{\sqrt{6}-\sqrt{2}}{4}+-i\frac{\sqrt{6}+\sqrt{2}}{4}]$$

$$\text{Essendo } \frac{19}{12}\pi=\frac{3}{2}\pi+\frac{\pi}{12} \text{ abbiamo } \cos(\frac{3}{2}\pi+\frac{\pi}{12})=\sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\text{e } \sin(\frac{3}{2}\pi+\frac{\pi}{12})=-\cos\frac{\pi}{12}=-\frac{\sqrt{6}+\sqrt{2}}{4} \text{ da cui la soluzione.}$$

2)

$$\sqrt[4]{-2-2\sqrt{3}i}$$

$$a = -2, \qquad b = -2\sqrt{3}, \qquad \varrho = \sqrt{4+12} = 4 \qquad \left.\begin{array}{l} \cos\theta = -\frac{1}{2} \\ \\ \sin\theta = -\frac{\sqrt{3}}{2} \end{array}\right\} \rightarrow \theta = \frac{4}{3}\pi$$

$$z_k = 4^{\frac{1}{4}}[\cos(\frac{\frac{4}{3}\pi + 2k\pi}{4}) + i\sin(\frac{\frac{4}{3}\pi + 2k\pi}{4})] \text{ per k = 0, 1, 2, 3}$$

$$z_0 = 4^{\frac{1}{4}}[\cos(\frac{1}{3}\pi + i\sin\frac{1}{3}\pi] = 4^{\frac{1}{4}}[\frac{1}{2} + i\frac{\sqrt{3}}{2}]$$

$$z_1 = 4^{\frac{1}{4}}[\cos(\frac{5}{6}\pi + i\sin\frac{5}{6}\pi] = 4^{\frac{1}{4}}[\cos(\frac{\pi}{6} - i\sin\frac{\pi}{6}] = 4^{\frac{1}{4}}[-\frac{\sqrt{3}}{2} + \frac{i}{2}]$$

$$z_2 = 4^{\frac{1}{4}}[\cos(\frac{4}{3}\pi + i\sin\frac{4}{3}\pi] = 4^{\frac{1}{4}}[-\cos(\frac{\pi}{3} + i(-\sin\frac{\pi}{3})] = 4^{\frac{1}{4}}[-\frac{1}{2} - i\frac{\sqrt{3}}{2}]$$

$$z_3 = 4^{\frac{1}{4}}[\cos(\frac{11}{6}\pi + i\sin\frac{11}{6}\pi] = 4^{\frac{1}{4}}[-\cos(\frac{\pi}{6} + i(-\sin\frac{\pi}{6})] = 4^{\frac{1}{4}}[\frac{\sqrt{3}}{2} - \frac{i}{2}]$$

Risolvere le seguenti equazioni con i numeri complessi:

1)

$$z^2 + 2z + 3 = 0$$

$$z = \frac{-1 \pm \sqrt{1-3}}{1}$$

$$z_1 = -1 - i\sqrt{2};\ z_2 = -1 + i\sqrt{2}$$

Notare che $\sqrt{-2} = \sqrt{-1 \cdot 2} = \sqrt{-1} \cdot \sqrt{2} = \sqrt{i^2} \cdot \sqrt{2} = i\sqrt{2}$

2)

$$z + 3i + Re(z)(i + (Im(z))^2) = 0$$

Sostituendo $z = a + ib$ abbiamo

$a+ib+3i+a(i+b^2) = 0 \rightarrow a+ib+3i+ai+ab^2 = 0 \rightarrow a+ib^2+i(b+3+a) = 0$

$$\begin{cases} a + ab^2 = 0 \\ b + 3 + a = 0 \end{cases} \rightarrow \begin{cases} a = 0 \quad \cup \quad 1 + b^2 = 0 \\ b + a = -3 \end{cases} \rightarrow \begin{matrix} \begin{cases} a = 0 \\ b = -3 \end{cases} \rightarrow z = -3i \\ \begin{cases} b = \sqrt{-1} \\ a + b = -3 \end{cases} \rightarrow \text{ nessuna soluzione reale} \end{matrix}$$

3)

$$z^2 + 2iz - 3 = 0$$

dato che il coefficiente di è b pari, usiamo la formula ridotta $z = \dfrac{-\frac{b}{2} \pm \sqrt{(\frac{b}{2})^2 - ac}}{a}$

$$z = -1 \pm \sqrt{i^2 + 3} = -i \pm \sqrt{1+3} \rightarrow \begin{matrix} z_1 = \sqrt{2} - i \\ z_2 = -\sqrt{2} - i \end{matrix}$$

4)

$$iz^3 = \overline{z}$$

Dato $z = \varrho(\cos\theta + i \sin\theta)$ abbiamo $z^3 = \varrho^3(\cos 3\theta + i \sin 3\theta)$ e $\overline{z} = \varrho(\cos\theta - i \sin\theta) \rightarrow$

$$i\varrho^3(\cos 3\theta + i\sin 3\theta) = \varrho(\cos\theta - i\sin\theta) \rightarrow$$
$$\varrho^3(i\cos 3\theta + i^2\sin 3\theta) = \varrho(\cos\theta - i\sin\theta) \rightarrow$$
$$\varrho^3(-\sin 3\theta + i\cos 3\theta) = \varrho(\cos\theta - i\sin\theta)$$

Definendo le condizioni per l'uguaglianza di due numeri complessi in forma trigonometrica abbiamo

$$\begin{cases} \varrho^3 = \varrho \\ \cos 3\theta = -\sin\theta \\ -\sin 3\theta = \cos\theta \end{cases} \rightarrow \begin{cases} \varrho^3 - \varrho = 0 \\ -\cot 3\theta = -\text{tg}\theta \end{cases} \rightarrow \begin{cases} \varrho = 0 \quad \cup \quad \varrho = \left\langle \begin{array}{l} +1 \\ -1 \text{ non ammissibile} \end{array} \right. \\ \text{tg}(\frac{\pi}{2} - 3\theta) = \text{tg}\theta \end{cases} \rightarrow$$

$$\begin{cases} \varrho = 0 \quad \cup \quad \varrho = 1 \\ \text{tg}(\frac{\pi}{2} - 3)\theta = \theta + k\pi \quad \text{ notare che tg}x = \text{tg}\beta \Leftrightarrow x = \beta + k\pi \end{cases} \rightarrow$$

$$\begin{cases} \varrho = 0 \vee \varrho = 1 \\ \frac{\pi}{2} - 4\theta = k\pi \leftrightarrow 4\theta = \frac{\pi}{2} - k\pi \leftrightarrow \theta = \frac{\pi - 2k\pi}{8} \quad \text{ per k = 0,1,2} \end{cases}$$

$$z_1 = 0$$
$$z_2 = \cos(\tfrac{\pi-2k\pi}{8}) + i\sin(\tfrac{\pi-2k\pi}{8}) \text{ per k = 0,1,2}$$

5)

$$z^6 + 2z^3 - 3 = 0$$

Ponendo $w = z^3$ abbiamo $w^2 + 2w - 3 = 0 z^3 = -1 \pm \sqrt{1+3} = \left\langle \begin{array}{l} z^3 = -3 \\ z^3 = 1 \end{array} \right.$

$$\text{Per } z^3 = -3 \ \varrho = 3$$
$$\left. \begin{array}{r} \cos\theta = \frac{-3}{3} \\ \sin\theta = 0 \end{array} \right\} \theta = \pi$$
$$z_1 = \sqrt[3]{3}(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$$
$$z_2 = \sqrt[3]{3}(\cos\pi + i\sin\pi)$$
$$z_3 = \sqrt[3]{3}(\cos\frac{5}{3}\pi + i\sin\frac{5}{3\pi})$$

$$\text{Per } z^3 = 1 \ \varrho = 1$$
$$\left. \begin{array}{r} \cos\theta = 1 \\ \sin\theta = 0 \end{array} \right\} \theta = 0$$
$$z_4 = \sqrt[3]{1}(\cos 0 + i\sin 0)$$
$$z_5 = \sqrt[3]{1}(\cos\frac{2}{3}\pi + i\sin 23\pi)$$
$$z_6 = \sqrt[3]{1}(\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi)$$

6)

$$z^2 + \overline{z} = 0$$

Sostituendo $z = a + ib$ abbiamo

$$(a+ib)^2 + a - ib = 0 \to a^2 - b^2 + 2aib + a - ib = 0$$

$$\begin{cases} a^2 - b^2 + a = 0 \\ 2ab - b = 0 \end{cases} \to \begin{cases} a^2 - b^2 + a = 0 \\ b(2a-1) = 0 \end{cases} \to$$

$$\begin{cases} a^2 - b^2 + a = 0 \\ b = 0 \quad \cup \quad a = \frac{1}{2} \end{cases} \to \begin{array}{l} \begin{cases} a^2 + a = 0 \\ b = 0 \end{cases} \to \begin{cases} a(a+1) = 0 \left\langle \begin{array}{l} a = -1 \\ a = 0 \end{array} \right. \\ b = 0 \end{cases} \to \begin{array}{l} z_1 = -1 \\ z_2 = 0 \end{array} \\ \begin{cases} (\frac{1}{2})^2 - b^2 + \frac{1}{2} = 0 \\ a = \frac{1}{2} \end{cases} \to \begin{cases} b^2 = \frac{3}{4} \left\langle \begin{array}{l} b = -\frac{\sqrt{3}}{2} \\ b = \frac{\sqrt{3}}{2} \end{array} \right. \\ a = \frac{1}{2} \end{cases} \to \begin{array}{l} z_3 = \frac{1}{2} - i\frac{\sqrt{3}}{2} \\ z_4 = \frac{1}{2} + i\frac{\sqrt{3}}{2} \end{array} \end{array}$$

7)

$$\overline{z}^4 = |z|$$

$$\varrho^4(\cos 4\theta - i \sin 4\theta) = \varrho$$

$$\begin{cases} \varrho^4 = \varrho \\ 4\theta = 2k\pi \end{cases} \to \begin{cases} \varrho(\varrho^3 - 1) = 0 \\ \theta = \frac{k\pi}{2} \end{cases} \to \begin{cases} \varrho = 0 \quad \cup \quad \varrho = 1 \\ \theta = \frac{k\pi}{2} \end{cases}$$

$$\begin{array}{l} z_0 = 0 \\ z_1 = \cos\frac{k\pi}{2} + i \sin\frac{k\pi}{2} \text{ per } k = 0,1,2,3 \end{array}$$

8)

$$iRe(z) + z^2 = |z|^2 + 1$$

Sostituendo $|z| = \sqrt{a^2+b^2}$ e $z = a + ib$ abbiamo

$$ai + a^2 - b^2 + 2aib - a^2 - b^2 - 1 = 0 \quad \to \quad a^2 - b^2 - a^2 - b^2 - 1 + i(a + 2ab) = 0$$

$$\begin{cases} a^2 - b^2 - a^2 - b^2 - 1 = 0 \\ a + 2b = 0 \end{cases} \to \begin{cases} -2b^2 - 1 = 0 \\ a(1+2b) = 0 \end{cases} \to \begin{cases} 2b^2 = -1 \\ a = 0 \quad \cup \quad b = -\frac{1}{2} \end{cases} \to$$

$$\begin{cases} b = \sqrt{-\frac{1}{2}} \\ a = 0 \quad \cup \quad b = -\frac{1}{2} \end{cases} \to \varnothing$$

9)

$$2|z|^2 = z^3$$

$$2\varrho^2 = \varrho^3(\cos 3\theta + i \sin 3\theta)$$

$$\begin{cases} 2\varrho^2 = \varrho^3 \\ 2k\pi = 3\theta \end{cases} \to \begin{cases} 2\varrho^2 - \varrho^3 = 0 \\ 2k\pi = 3\theta \end{cases} \to \begin{cases} \varrho^2(2-\varrho) = 0 \left\langle \begin{array}{l} \varrho = 0 \\ \varrho = 2 \end{array} \right. \\ \theta = \frac{2k\pi}{3} \text{ per } k = 0,1,2 \end{cases}$$

$$\begin{array}{l} z_1 = 0 \\ z_2 = 2(\cos\frac{2k\pi}{3} + i \sin\frac{2k\pi}{3}) \quad \text{ per } k = 0,1,2 \end{array}$$

10)

$$z^2 + im(z) + 2\overline{z} = 0 \text{ dove z = a + ib}$$

$$\Leftrightarrow (a+ib)^2 + ib + 2(a-ib) = 0$$

$$\Leftrightarrow a^2 - b^2 + 2iab + ib - 2a - 2ib = 0$$

$$\Leftrightarrow a^2 - b^2 + 2a + i(2ab - b) = 0$$

$$\Leftrightarrow \begin{cases} a^2 - b^2 + 2a = 0 \\ 2ab - b = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} a^2 - b^2 + 2a = 0 \\ b(2a-1) = 0 \end{cases} \Leftrightarrow a = \frac{1}{2} \quad \cup \quad b = 0$$

Se $a = \frac{1}{2} \rightarrow \frac{1}{4} - b^2 + 1 = 0 \Rightarrow b = \pm\frac{\sqrt{5}}{2} \Rightarrow \qquad z_1 = \frac{1}{2} - i\frac{\sqrt{5}}{2}, \quad z_2 = \frac{1}{2} + i\frac{\sqrt{5}}{2}$

Se $b = 0 \rightarrow \quad a^2 + 2b = 0 \Rightarrow \quad a = 0 \quad a = -2 \Rightarrow \qquad z_3 = 0, \quad z_4 = -2$