COVERINGS OF COMMUTATORS IN PROFINITE GROUPS

Cristina Acciarri *
Department of Mathematics
University of Brasilia, Brazil
E-mail: acciarricristina@yahoo.it

By a result of Baer, when an abstract group G is covered by finitely many cyclic subgroups, then G is either cyclic or finite. This suggests the general idea that if w is a group word, for certain choices of w, and the set of all w-values in G is contained in the union of finitely many cyclic subgroups, then the verbal subgroup $w(G)$ should be not too far away from being finite or cyclic.

In a more general setting if the set of all w-values in G is covered by finitely, or countably, many subgroups with certain specific properties, then one should hope that the structure of the corresponding verbal subgroup $w(G)$ is somehow similar to that of the covering subgroups.

In this talk we will present several recent results that illustrate this phenomenon in the realm of abstract and profinite groups.

ON NORMALIZED INTEGRAL TABLE ALGEBRAS GENERATED BY A FAITHFUL NON-REAL ELEMENT OF DEGREE 3 WITH AN ELEMENT OF DEGREE 2

Zvi Arad
Bar Ilan University and Netanya Academic College, Israel
E-mail: aradtzvi@macs.biu.ac.il

It is well known that a perfect group G has no nontrivial linear irreducible character. By results of H. F Blichfeldt, H. Blau, Z. Arad, Muhamad Awais and Chen Guiyun proved the following:

Lemma. *Let G be a perfect finite group with a faithful irreducible character of degree 3. Then G has no irreducible character of degree 2.*

This inspired Arad, Awais and Chen to state the following conjecture:

Conjecture. *Let (A, B) be a Normalized Integral Table algebra generated by a faithful non real element b_3 in B of degree 3. Assume that $L_1(B) = 1$. Then $L_2(B)$ is the empty set. Here $L_i(B)$ is the set of elements of degree i in B.*

In our research we proved the conjecture under additional conditions.

If one can prove the conjecture then we can eliminate the assumption that $L_2(B)$ is the empty set in the Main Theorem of [1].

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* Joint work with Pavel Shumyatsky.

ON METACYCLIC SUBGROUPS OF FINITE GROUPS

Adolfo Ballester-Bolinches
Departament d'Àlgebra
Universitat de València, Spain
E-mail: Adolfo.Ballester@uv.es

An important part of finite group theory is the recovery of global information about a group from local information. Many results derive global information from knowledge of the structure of a restricted class of subgroups.

This talk is intended to present some contributions in this context, and it is organised around two questions concerning the influence of metacyclic Sylow subgroups and 2-generator subgroups on the structure of a finite group.

FREE SUBGROUPS IN GROUP RINGS

Victor Bovdi
Department of Mathematical Sciences
UAEU, U.A.E.
E-mail: vbovdi@gmail.com

Let $V(DG)$ be the group of normalized units of the group ring DG of a group G over the (commutative) integral domain D of characteristic zero.

If G is a finite non-Dedekind group, then B. Hartley and P. F. Pickel [4] proved that $V(DG)$ contains a free subgroup of rank 2. If x is a nilpotent element of the integral group ring $\mathbb{Z}G$ and $*$ is the classical involution, then A. Salwa [5] showed that noncommuting unipotent elements $\{1+x, 1+x^*\}$ generate a free subgroup of rank 2.

In [2] we introduced a new family of torsion and non-torsion units in $V(DG)$. Suppose that G is non-Dedekind and contains a non-normal finite cyclic subgroup of order n. Using these units we prove that the group ring DG contains the free product $C_n \star C_n$ as a subgroup, and this subgroup is normally generated by a single element.

Note that several problems in group theory and small dimensional topology can be reduced to the question whether a given group can be normally generate by a single element.

For some classes of groups we provide a simple alternative proof of the above result by B. Hartley and P. F. Pickel, and improve it by showing that the free subgroup $C_\infty \star C_\infty$ also can be normally generated by a single element.

Furthermore we will demonstrate how to apply the results of [1, 3] to reveal the structure of $V(DG)$.

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GROUPS IN WHICH EVERY SUBGROUP IS COMMENSURABLE TO A SUBNORMAL SUBGROUP

Carlo Casolo
Dipartimento di Matematica e Informatica "U. Dini"
Università di Firenze, Italy
E-mail: ccasolo@unifi.it

By an almost trivial argument, a group G belongs to the class in the title if and only if every subgroup of it is a finite extension of a subgroup that is subnormal in G. This class was considered by H. Heineken [1] some twenty years ago, but, while the interesting special case in which every subgroup is normal-by-finite, introduced in [2], has been the object of several papers, it seems that not much has been since added to Heineken's results for the larger class. Our purpose is, building on those results of Heineken, to contribute in this sense. The single most (and perhaps only) interesting result is the following

Theorem. *A locally finite group in which every subgroup is subnormal-by-finite is nilpotent-by-Černikov.*

We will then discuss possible extensions (and what makes them complicate in general), as well as the effect of adding bounds on the index of the 'subnormal cores', or on their defects.

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GROUPS IN WHICH EACH SUBGROUP IS COMMENSURABLE TO A NORMAL ONE *

Ulderico Dardano **
Dipartimento di Matematica e Applicazioni "R.Caccioppoli"
Università di Napoli Federico II, Italy
E-mail: dardano@unina.it

A group G is said a CN-group if and only if for each $H \leq G$ there is $N \lhd G$ such that $|HN : (H \cap N)|$ is finite. The class of CN-groups contains properly both the class of groups G such that $\forall H \leq G \ \ |H^G : H| < \infty$ and the class of CF-groups G such that $\forall H \leq G \ \ |H : H_G| < \infty$ (CF-groups, core-finite).

Groups in the former class are exactly the finite-by-abelian groups (see [2]). Dually, CF-groups are abelian-by-finite, provided all their periodic quotient are locally finite (see [1] and [3]). On the other hand Tarski monsters are CF.

Our main result is the following:

Theorem. *Let G be a group in which every periodic image is locally finite. If G is CN, then G is finite-by-abelian-by-finite.*

Further more detailed results on the topic will be discussed in the talk.

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* This talk is dedicated to the memory of Mario Curzio.
** Joint work with Carlo Casolo and Silvana Rinauro.

FROM FINITE TO INFINITE: A STRATEGY FOR GROUPS OF HIGH CARDINALITY

Francesco de Giovanni
Dipartimento di Matematica e Applicazioni
Università di Napoli Federico II, Italy
E-mail: degiovan@unina.it

It is well-known that finite groups are strongly influenced by their arithmetic properties; obvious evidence of this phenomenon is given for instance by the classical theorems of Lagrange, Sylow, Burnside and Feit-Thompson. Clearly, results of this type are no longer available in the infinite case. On the other hand, it is possible to show that suitable conditions on the size of a group G of high cardinality (and on that of its subgroups) restrict the structure of G, at least with respect to the most relevant embedding properties.

THE RELATIONSHIP BETWEEN THE UPPER AND LOWER CENTRAL SERIES: A SURVEY

Martyn R. Dixon *
Department of Mathematics
University of Alabama
Tuscaloosa, AL. 35487-0350 U.S.A.
E-mail: mdixon@ua.edu

Let

$$1 = Z_0(G) \leq Z_1(G) = Z(G) \leq Z_2(G) \leq \cdots \leq Z_\alpha(G) \leq \ldots Z_\gamma(G)$$

be the upper central series of G and let

$$G = \gamma_1(G) \geq \gamma_2(G) = G' \geq \cdots \geq \gamma_\alpha(G) \ldots$$

be the lower central series of G.

The following theorem is fundamental in group theory:

Theorem. *Let G be a group and let k be a natural number. If $G/Z_k(G)$ is finite then $\gamma_{k+1}(G)$ is also finite.*

In this talk I will discuss various generalizations of this theorem. A group G is called *generalized radical* if it has an ascending series whose factors are either locally nilpotent or locally finite. If p is a prime, then a group G has *finite section p-rank $r_p(G) = r$* if every elementary abelian p-section of G is finite of order at most p^r and there is an elementary abelian p-section A/B of G such that $|A/B| = p^r$.

Theorem. *Let G be a locally generalized radical group and let p be a prime. Suppose that $G/Z_k(G)$ has finite section p-rank at most r. Then $\gamma_{k+1}(G)$ has finite section p-rank. Moreover there exists a function $\tau(r, k)$ such that $r_p(\gamma_{k+1}(G)) \leq \tau(r, k)$.*

* Joint work with Leonid Kurdachenko and Javier Otal.

SOME QUESTIONS SUGGESTED BY FITTING'S THEOREM

Martin J. Evans *
Department of Mathematics
The University of Alabama, U.S.A.
E-mail: mevans@ua.edu

Let M and N be normal subgroups of a group G and suppose that they are nilpotent of class c and class d, respectively. Fitting's theorem asserts that MN is nilpotent of class at most $c+d$. It follows that if a nilpotent group G has distinct maximal subgroups M and N, each of class n, then G has class at most $2n$.

Suppose now that G is a nilpotent group in which *all* proper subgroups have class at most n. It follows easily from the discussion above that G has class at most $2n$. Moreover, examples exist of groups G of this sort that have class (exactly) $2n$ (at least for $n = 3$, see [2]) but their structure is necessarily very restricted. For instance, such a group G must be 2-generator [1].

In this talk we'll consider the possibility of obtaining similar results for torsion-free nilpotent groups in which all subgroups *of infinite index* have class at most n. We'll also consider some related questions for other types of algebraic structures.

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* Joint work with Bryan G. Sandor.

BEAUVILLE STRUCTURES IN FINITE p-GROUPS

Gustavo A. Fernández-Alcober *
University of the Basque Country UPV/EHU
Bilbao (Spain)
E-mail: gustavo.fernandez@ehu.eus

A Beauville surface of unmixed type is a compact complex surface which is the quotient of the product of two algebraic curves by a special action of a finite group. The groups that can appear in such a construction are called *Beauville groups*. It is easy to give a purely group theoretical description of Beauville groups. If G is a group and $x, y \in G$, let $\Sigma(x, y)$ be the union of all conjugates of $\langle x\rangle$, $\langle y\rangle$ and $\langle xy\rangle$. Then G is a Beauville group if and only if it is a 2-generator group having two systems of generators $S_1 = \{x_1, y_1\}$ and $S_2 = \{x_2, y_2\}$ such that $\Sigma(x_1, y_1) \cap \Sigma(x_2, y_2) = 1$. If that is the case, we say that S_1 and S_2 form a *Beauville structure* for G.

Beauville groups have attracted much interest in recent years. In 2000, Catanese showed that the abelian Beauville groups are those of the form $C_n \times C_n$ with $\gcd(n, 6) = 1$. On the other hand, Guralnick and Malle proved in 2012 that any non-abelian finite simple group other than A_5 is a Beauville group. However, only scarce information was known regarding finite p-groups.

In this talk, we first present a criterion for a 2-generator p-group G with a "nice power structure" to be a Beauville group: if $\exp G = p^e$ then G is a Beauville group if and only if $|G^{p^{e-1}}| \geq p^2$. This shows that Catanese's criterion can be extended to this class of p-groups, which covers all the known families of well-behaved p-groups with respect to powers: regular p-groups (in particular groups of order $\leq p^p$), powerful p-groups and more generally potent p-groups, and p-central p-groups. Our results show how to obtain easily Beauville structures in these groups.

Then we characterize which quotients of the Nottingham group over $\mathbb{F}_p$ are Beauville groups. As a consequence, we determine the first explicit family of Beauville 3-groups in the literature.

Finally, we study the existence of Beauville structures in metabelian thin p-groups, including groups of maximal class. In this case, the criterion for groups with nice power structure is not valid any more, but we are able anyway to solve the problem in terms of natural invariants of the group. This last part is joint work with Norberto Gavioli and Carlo Scoppola, from the University of L'Aquila (Italy).

* Joint work with Sükran Gül (Middle East Technical University, Ankara, Turkey.)

FINITE GROUPS WITH 6 AUTOMORPHISM ORBITS

Martino Garonzi *
Department of Mathematics
University of Brasilia, Brazil
E-mail: mgaronzi@gmail.com

For G a group denote by $\omega(G)$ the number of "automorphism orbits" of G, that is, the number of orbits of the natural action of $\mathrm{Aut}(G)$ on G. It is interesting to ask what can we say about G only knowing $\omega(G)$. It is obvious that $\omega(G) = 1$ if and only if $G = \{1\}$, and it is well-known that if G is a finite group then $\omega(G) = 2$ if and only if G is elementary abelian. Laffey and MacHale in [1] showed that the alternating group A_5 (which has $\omega(A_5) = 4$) is the only finite nonsolvable group G with $\omega(G) \leq 4$, and Zhang in [3] classified the finite groups in which any two elements of the same order lie in the same automorphism orbit. Stroppel in [2] showed that the only finite nonabelian simple groups G with $\omega(G) \leq 5$ are the groups $PSL(2, \mathbb{F}_q)$ with $q \in \{4, 7, 8, 9\}$, and he proposed the following problem (Problem 2.5): classify all the finite nonsolvable finite groups G with $\omega(G) \leq 6$.

In this talk we propose our solution to Stroppel's problem. Our result is the following:

Theorem. *If G is a finite nonsolvable group with $\omega(G) \leq 6$ then G is isomorphic to one of $PSL(2, \mathbb{F}_q)$ with $q \in \{4, 7, 8, 9\}$, $PSL(3, \mathbb{F}_4)$ or $ASL(2, \mathbb{F}_4)$.*

Here $ASL(2, \mathbb{F}_4)$ is the affine group ${\mathbb{F}_4}^2 \rtimes SL(2, \mathbb{F}_4)$ where $SL(2, \mathbb{F}_4)$ acts naturally on ${\mathbb{F}_4}^2$.

We also display an infinite family of nonsolvable groups G with $\omega(G) = 7$.

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* Joint work with Raimundo Bastos, Alex Carrazedo Dantas.

GROUPS, AUTOMATA AND AUTOMATICALLY GENERATED SEQUENCES

Rostislav Grigorchuk
Texas A&M University, Department of Mathematics
College Station, TX 77843-3368, U.S.A.
E-mail: grigorch@math.tamu.edu

After a short survey on the use of finite automata of different types in group theory I will focus on two topics:

1. embedding into topological full groups of minimal subshifts (including a relation to the random Schroedinger operator);
2. Jacobi type representations, automatic representations and unitriangular automatic representations. The role in all of this of automatically generated sequences will be explain as well.

CYCLES AND BIPARTITE DIVISOR GRAPH ON IRREDUCIBLE CHARACTER DEGREES

Roghayeh Hafezieh
Department of Mathematics
Gebze Technical University, Turkey
E-mail: roghayeh@gtu.edu.tr

Let G be a finite group. We consider the set of irreducible complex characters of G, namely $Irr(G)$, and the related degree set $cd(G) = \{\chi(1) : \chi \in Irr(G)\}$. Let $\rho(G)$ be the set of all primes which divide some character degree of G. In [3], Taeri considered the case that the bipartite divisor graph of the set of conjugacy class sizes is a cycle and proved that for a finite nonabelian group G, the bipartite divisor graph of the conjugacy class sizes is a cycle if and only if it is a cycle of length 6, and for an abelian group A and $q \in \{4, 8\}$, $G \simeq A \times SL_2(q)$. Inspired by this paper, in this work we consider the bipartite divisor graph for the set of irreducible character degrees of a finite group and define it as follows:

Definition. *Let G be a finite group. The bipartite divisor graph for the set of irreducible character degrees of G, is an undirected bipartite graph with vertex set $\rho(G) \cup (cd(G) \setminus \{1\})$, such that an element p of $\rho(G)$ is adjacent to an element m of $cd(G) \setminus \{1\}$ if and only if p divides m. We denote this graph simply by $B(G)$.*

We prove that $B(G)$ is a cycle if and only if G is solvable and $B(G)$ is a cycle of length four or six. By using these properties, we prove if $B(G)$ is a cycle of length four, then there exists a normal abelian Hall subgroup of G which explains the structure of the irreducible character degrees of G.

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THE GROTHENDIECK GROUP AS A CLASSIFICATION TOOL FOR ALGEBRAS

Roozbeh Hazrat
Western Sydney University, Australia
E-mail: R.Hazrat@westernsydney.edu.au

From a graph (e.g., cities and flights between them) one can generate an algebra which captures the movements along the graph One such algebras are Leavitt path algebras.

Despite being introduced only 10 years ago, Leavitt path algebras have arisen in a variety of different contexts as diverse as analysis, symbolic dynamics, noncommutative geometry and representation theory. In fact, Leavitt path algebras are algebraic counterpart to graph C*-algebras, a theory which has become an area of intensive research globally. There are strikingly parallel similarities between these two theories. Even more surprisingly, one cannot (yet) obtain the results in one theory as a consequence of the other; the statements look the same, however the techniques to prove them are quite different (as the names suggest, one uses Algebra and other Analysis). These all suggest that there might be a bridge between Algebra and Analysis yet to be uncovered.

In this talk, we introduce Leavitt path algebras and try to classify them by means of (graded) Grothendieck groups. We will ask nice questions!

RECENT GENERALIZATIONS OF THE BERKOVICH-CHILLAG-HERZOG THEOREM

Marcel Herzog
Tel Aviv University, Israel
E-mail: herzogm@post.tau.ac.il

In 1992, Berkovich, Chillag and Herzog characterized finite groups with all non-linear irreducible characters having distinct degrees (BCH-paper). In this talk I shall describe three generalizations of this result. In 2007, Maria Loukaki characterized solvable groups G satisfying the following property: there exists a non-trivial normal subgroup N of G such that all irreducible characters of G which do not contain N in their kernel are of distinct degrees. The solvable version of the BCH-paper is obtained by taking $N = G$". The second result is a 2016-paper of Silvio Dolfi and Manoj Yadav. They classify finite groups whose non-linear characters that are not conjugate under the natural Galois action, have distinct degrees. The third result is a paper in preparation of Mariagrazia Bianchi, Emanuele Pacifici and myself. In this paper we consider finite groups with non-trivial intersections of kernels of all but one irreducible characters. In particular, we characterize their subclass, which properly contains all finite groups considered in the BCH-paper.

PRO-p METHODS IN THE THEORY OF FINITE p-GROUPS

Andrei Jaikin-Zapirain
Departamento de Matemáticas, Universidad Autónoma de Madrid, and
Instituto de Ciencias Matemáticas - CSIC, UAM, UCM, UC3M.
28049 Madrid, Spain.
E-mail: andrei.jaikin@uam.es

I will explain how pro-p techniques can help to answer some questions about finite p-groups. The origin of pro-p methods lies in Leedham-Green's solution of the coclass conjectures.

In my talk I will focuse on the following two new applications of pro-p techniques.

Theorem. *([2]) Let k be an odd natural number.*

1. *If $k < 24$, then there is only a finite number of finite 2-groups with exactly k real conjugacy classes.*
2. *If $k = 25$, then there are infinitely many finite 2-groups with exactly k real conjugacy classes.*

Theorem. *([1]) For each prime p there exists a non-abelian finite p-group G such that $|G|$ does not divide $|Aut(G)|$.*

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ON THE NONABELIAN TENSOR PRODUCT OF CYCLIC GROUPS OF p-POWER ORDER

Luise-Charlotte Kappe
Binghamton University, U.S.A.
E-mail: menger@math.binghamton.edu

The non-abelian tensor product of a pair of groups was introduced by R. Brown and J.-L. Loday. It arises in the applications in homotopy theory of a generalized Van Kampen theorem.

Let G and H be groups which act on each other via automorphisms and which act on themselves via conjugation. The actions are said to be compatible if ${}^{{}^g h}g' = {}^{g}({}^{h}({}^{g^{-1}}g'))$ and ${}^{{}^h g}h' = {}^{h}({}^{g}({}^{h^{-1}}h'))$ for all $g, g' \in G$ and $h, h' \in H$. The nonabelian tensor product $G \otimes H$ is defined provided G and H act compatibly. In such a case $G \otimes H$ is the group generated by the symbols $g \otimes h$ with relations $gg' \otimes h = ({}^g g' \otimes {}^g h)(g \otimes h)$ and $g \otimes hh' = (g \otimes h)({}^h g \otimes {}^h h')$ for $g, g' \in G$ and $h, h' \in H$.

If $G = H$, we call $G \otimes G$ the tensor square of G. Here the action is conjugation which is always compatible. Good progress has been made in determining the nonabelian tensor square for large classes of groups.

However in the case of nonabelian tensor products the enigma of compatible actions has prevented such progress. Only in a few cases the nonabelian tensor product of two groups with nontrivial compatible actions has been determined. One such case is the nonabelian tensor product of two infinite cyclic groups, where the mutual actions are inversion. In 1989 Gilbert and Higgins showed that the nonabelian tensor product was isomorphic to the free abelian group of rank 2, contradicting an earlier conjecture that the nonabelian tensor product of two cyclic groups is cyclic.

We were able to show that the minimal number of generators of a nonabelian tensor product of two cyclic groups does not exceed two. Furthermore, we established a necessary and sufficient condition that a pair of actions on two cyclic groups is a compatible pair. With its help we classified all compatible actions in the case of cyclic p-groups. The resulting nonabelian tensor products turn out to be cyclic p-groups with the exception of some 2-groups with certain actions of order 2.

This is joint work with M.P. Visscher and M.S. Mohamad.

REMARKS ON UNCOUNTABLE SIMPLE GROUPS

Otto Kegel
University of Freiburg, Germany
E-mail: kegel@email.mathematik.uni-freiburg.de

Every infinite simple group G has as a local system consisting of countably infinite simple subgroups.

ALMOST ENGEL FINITE AND PROFINITE GROUPS

Evgeny Khukhro *
School of Mathematics and Physics, University of Lincoln, U.K.,
and Sobolev Institute of Mathematics, Novosibirsk, Russia
E-mail: khukhro@yahoo.co.uk

Let g be an element of a group G. For a positive integer n, let $E_n(g)$ be the subgroup generated by all commutators $[...[[x, g], g], \dots, g]$ over $x \in G$, where g is repeated n times. We prove that if G is a profinite group such that for every $g \in G$ there is $n = n(g)$ such that $E_n(g)$ is finite, then G has a finite normal subgroup N such that G/N is locally nilpotent. The proof uses the Wilson–Zelmanov theorem saying that Engel profinite groups are locally nilpotent. In the case of a finite group G, we prove that if, for some n, we have $|E_n(g)| \leq m$ for all $g \in G$, then the order of the nilpotent residual $\gamma_\infty(G)$ is bounded in terms of m.

GENERATORS FOR DISCRETE SUBGROUPS OF 2-BY-2 MATRICES OVER RATIONAL QUATERNION ALGEBRAS

Ann Kiefer
Department of Mathematics
Vrije Universiteit Brussel, Belgium & Universität Bielefeld, Germany
akiefer@vub.ac.be

In [1], we developed an algorithm to determine generators for discrete subgroups of quaternion algebras over quadratic imaginary extensions of $\mathbb{Q}$ or discrete subgroups of 2-by-2 matrices over quadratic extensions of $\mathbb{Q}$. These groups act discontinuously on hyperbolic 3-space and the algorithm constructs a fundamental domain to find a set of generators.

In this work we generalize this algorithm to 2-by-2 matrices over the group of invertible elements of a Clifford algebra. The group of such matrices acts discontinuously on an n-dimensional hyperbolic space.

Via an exceptional isomorphism, this gives generators of 2-by-2 matrices over the rational quaternion algebra.

*Joint work with Pavel Shumyatsky.

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SYLOW NUMBERS AND CHARACTER TABLES

Iris Köster *
Fakultät Mathematik und Physik
University of Stuttgart, Germany
E-mail: iris.koester@mathematik.uni-stuttgart.de

Let G be a finite group and let $n_p(G)$ denote the number of Sylow p-subgroups of G. The set $\mathrm{sn}(G)$ of all Sylow numbers $n_p(G)$ is called Sylow numbers of G.

In 2003 G. Navarro gave an overview of open questions concerning Sylow subgroups and characters [2]. Among these open problems he posed the question whether the character table determines the Sylow numbers. For solvable groups I. M. Isaacs and G. Navarro proved that the character table provides the primes dividing the normalizer $|\mathrm{N}_G(P)|$ of a Sylow p-subgroup P in G [3].

In this talk we want to present a different approach to this problem. We were able to give a affirmative answer for finite metanilpotent and supersolvable groups. Furthermore we could prove that in order to solve this problem we only need to consider groups with exactly one minimal normal subgroup. In the last part of the talk we will show that the integral group ring $\mathbb{Z}G$ of a finite solvable group G determines the Sylow numbers of G.

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* Joint work with Wolfgang Kimmerle.

ON INFLUENCE OF SUBGROUP FAMILIES, WHICH HAVE SOME FINITE RANKS, ON THE STRUCTURE OF GROUPS

Leonid Kurdachenko
Dnepropetrovsk National University, Ukraine
E-mail: khukhro@yahoo.co.uk

The talk is devoted to consideration of the influence some important families of subgroups on the structure of group. One of typical result is following:

Let G be a locally generalized radical group. If every locally (soluble and minimax) subgroup of G has finite special rank, then G has finite special rank.

HOMOGENEOUS MONOMIAL GROUPS *

Mahmut Kuzucuoğlu **
Department of Mathematics
Middle East Technical University, Ankara, Turkey
E-mail:matmah@metu.edu.tr

Basic properties of monomial groups are studied by O. Ore in [2] including the structure of the centralizers of elements. D-Hyperoctahedral groups are defined and the lattice of normal subgroups, imprimitivity system are studied and the classification of these groups by using the lattice of Steinitz numbers is given in [1]. Here we define the homogeneous monomial groups and find the structure of the centralizers of elements in these groups. D-Hyperoctahedral groups are special case of homogeneous monomial groups when H is isomorphic to the cyclic group of order two.

Let H be a group and denote the complete monomial group of degree m over H by $\Sigma_m(H)$. Let $\xi = (p_1, p_2, \ldots)$ be an infinite sequence of not necessarily distinct primes. The homogeneous monomial group with respect to the above sequence is denoted by $\Sigma_\xi(H)$ and it is obtained as a direct limit of the groups $\Sigma_{n_i}(H)$ embedded into $\Sigma_{n_{i+1}}(H)$ by strictly diagonal embeddings where $n_i = p_1 p_2 \ldots p_i$.

Theorem. *(Kuzucuoğlu, Oliynyk, Sushchanskii) Let ρ be an element of $\Sigma_\xi(H)$ with principal beginning is in $\Sigma_{n_k}(H)$ with its normal form $\rho = \lambda_1 \ldots \lambda_l$, where $\lambda_i = \gamma_{i1} \ldots \gamma_{ir_i}$ where for a fixed i the γ_{ij} are the normalized cycles of the same length m_i and the determinant class a_i. Then the centralizer $C_{\Sigma_\xi(H)}(\rho) \cong C_{a_1}(\Sigma_{\xi_1}(C_{a_1})) \times C_{a_2}(\Sigma_{\xi_2}(C_{a_2})) \times \ldots \times C_{a_l}(\Sigma_{\xi_l}(C_{a_l}))$ where C_{a_i} is the centralizer of a single element γ_{ij} in $\Sigma_{m_i}(H)$. The group C_{a_i} consists of elements of the form $\kappa = [c_i]\gamma_{i1}^j$ where the element c_i belongs to the group $C_H(a_i)$ and $Char(\xi_i) = \frac{Char(\xi)}{n_k} r_i$.*

Theorem. *(Kuzucuoğlu, Oliynyk, Sushchanskii) Let ξ and ν be two infinite sequences of prime numbers. The homogenous monomial groups $\Sigma_\xi(H)$ and $\Sigma_\nu(G)$ are isomorphic if and only if $G \cong H$ and $Char(\xi) = Char(\nu)$.*

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* This talk is dedicated to Guido Zappa.

** Joint work with B. V. Oliynyk, V. I. Sushchanskii.

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GROUPS EQUAL TO A PRODUCT OF THREE CONJUGATE SUBGROUPS

Dan Levy
The School of Computer Sciences
The Academic College of Tel-Aviv-Yaffo, Israel
E-mail: danlevy@mta.ac.il

Let G be a finite non-solvable group. We prove that there exists a proper subgroup A of G such that G is the product of three conjugates of A, thus replacing an earlier upper bound of 36 with the smallest possible value. The proof relies on an equivalent formulation in terms of double cosets, and uses the following theorem which is of independent interest and wider scope: Any group G with a BN-pair and a finite Weyl group W satisfies $G = (Bn_0B)^2 = BB^{n_0}B$ where n_0 is any preimage of the longest element of W.

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BRAUER CHARACTERS OF q'-DEGREES

Mark L. Lewis *
Department of Mathematical Sciences
Kent State University, United States
E-mail:

We show that if p is a prime and G is a finite p-solvable group satisfying the condition that a prime q divides the degree of no irreducible p-Brauer character of G, then the normalizer of some Sylow q-subgroup of G meets all the conjugacy classes of p-regular elements of G.

* Joint work with Hung Tong Viet.

THE CHEBOTAREV INVARIANT OF A FINITE GROUP

Andrea Lucchini
Dipartimento di Matematica
Universit di Padova, Italy
E-mail: lucchini@math.unipd.it

We say that a subset $\{g_1, \dots, g_d\}$ of a finite group G invariably generates G if $\{g_1^{x_1}, \dots, g_d^{x_d}\}$ generates G for every choice of $x_i \in G$. The Chebotarev invariant $C(G)$ of G is the expected value of the random variable n that is minimal subject to the requirement that n randomly chosen elements of G invariably generate G. The main motivation for introducing the invariant $C(G)$ is the relationship to Chebotarev's Theorem and the calculation of Galois groups of polynomials with integer coefficients. Chebotarev's Theorem provides elements of a suitable Galois group G, where the elements are obtained only up to conjugacy in G; the interest in the study of $C(G)$ comes from computational Galois theory, where there is a need to know how long one should expect to wait in order to ensure that choices of representatives from the conjugacy classes provided by Chebotarev's Theorem will generate G.

In response to a question of Kowalski and Zywina [2], Kantor, Lubotzky and Shalev [1] bounded the size of a randomly chosen set of elements of G that is likely to generate G invariably. As a corollary of their result, they proved that there exists an absolute constant c such that $C(G) \leq c\sqrt{|G| \log |G|}$ for all finite groups G. This bound is close to best possible: sharply 2-transitive groups provide an infinite family of groups G for which $C(G) \sim \sqrt{|G|}$. In fact Kowalski and Zywina ask whether $C(G) = O(\sqrt{|G|})$ for all finite groups G. We give an affirmative answer [3]:

Theorem. *There exists an absolute constant β such that $C(G) \leq \beta\sqrt{|G|}$ for all finite groups G.*

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SUBGROUP ISOMORPHISM PROBLEM AND SYLOW THEOREMS FOR UNITS IN INTEGRAL GROUP RINGS

Leo Margolis
IAZ
University of Stuttgart, Germany
leo.margolis@mathematik.uni-stuttgart.de

Let G be a finite group, $\mathbb{Z}G$ the integral group ring of G and denote by $\mathrm{V}(\mathbb{Z}G)$ the group of normalized units in $\mathbb{Z}G$, i.e. those units whose coefficients sum up to 1. In his PhD-thesis from 1940 G. Higman asked, whether any finite subgroup of $\mathrm{V}(\mathbb{Z}G)$ is isomorphic to a subgroup of G. As in 1997 this turned out to be wrong in general the following question arises:

Subgroup Isomorphism Problem (SIP): For which finite group U does the following statement hold: If $\mathrm{V}(\mathbb{Z}G)$ contains a group isomorphic to U for some finite group G, then G contains a subgroup isomorphic to U.

The only known counterexamples for (SIP) are groups arising from M. Hertweck's counterexample to the Isomorphism Problem, in particualr these groups are bigger then the Monster. However the only groups for which (SIP) is known are cyclic groups of prime power order and elementary-abelian groups of rank 2. I will report on a proof of (SIP) for $C_4 \times C_2$ and related Sylow Theorems for $\mathrm{V}(\mathbb{Z}G)$, i.e. whether Higman's question might have a positive answer for p-subgroups.

ON THE COVERING NUMBER OF SMALL SYMMETRIC GROUPS AND SOME SPORADIC SIMPLE GROUPS

Daniela Nikolova-Popova *
Department of Mathematical Sciences
Florida Atlantic University, U.S.A.
E-mail: dpopova@fau.edu

We say that a group G has a finite covering if G is a set theoretical union of finitely many proper subgroups. The minimal number of subgroups needed for such a covering is called the covering number of G denoted by $\sigma(G)$.

Let S_n be the symmetric group on n letters. For odd n Maroti determined $\sigma(S_n)$ with the exception of $n = 9$, and gave estimates for n even showing that $\sigma(S_n) \leq 2n - 2$. Using GAP calculations, as well as incidence matrices and linear programming, we show that $\sigma(S_8)$ $= 64$, $\sigma(S_{10}) = 221$, $\sigma(S_{12}) = 761$. We also show that Maroti's result for odd n holds without exception proving that $\sigma(S_9) = 256$. We establish in addition that the Mathieu group M_{12} has covering number 208, and improve the estimate for the Janko group J_1 given by P.E. Holmes.

* Joint work with Luise-Charlotte Kappe, Eric Swartz.

A GROUP THEORETICAL PROBLEM INSPIRED BY THE ČERNÝ CONJECTURE

Péter P. Pálfy
Alfréd Rényi Institute of Mathematics,
Hungarian Academy of Sciences
and Eötvös University, Budapest
E-mail: ppp@renyi.hu

The Černý conjecture is a half-century-old famous unsolved problem in automata theory. In 1964 the Slovakian computer scientist Ján Černý [1] conjectured than for any synchronizing automaton with n states there exists a synchronizing word of length at most $(n-1)^2$. In common mathematical language that means if the composition of some given transformations on the n-element set is a constant map, then one can find a suitable composition of these transformations with at most $(n-1)^2$ factors that also gives a constant map. The conjecture has been proved for various classes of automata, but it is still open in general. The best known upper bound is $(n^3-n)/6$, due to Jean-Éric Pin [5] and Péter Frankl [2]. I will present the ingenious combinatorial proof by Frankl leading to this result. A possible idea for improving this bound inspired the following group theoretic question.

Problem. *Let $g_1, \ldots, g_k$ generate a transitive permutation group on the n-element set Ω. Let $A, B \subset \Omega$, and assume that $|A| \cdot |B| < |\Omega|$. Then there exists a permutation $g \in \langle g_1, \ldots, g_k \rangle$ of word length at most $|A| \cdot |B|$ such that $Ag \cap B = \emptyset$.*

There is a large variety of examples for which we cannot do better than $|A| \cdot |B|$, but to prove that this is a general upper bound seems to be a hard problem.

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ALGEBRAS AND REGULAR SUBGROUPS

Marco Antonio Pellegrini *
Dipartimento di Matematica e Fisica
Università Cattolica del Sacro Cuore, Brescia, Italy
E-mail: marcoantonio.pellegrini@unicatt.it

In this talk we will describe the results obtained in [4] concerning the subgroups R of the affine group $\mathrm{AGL}_n(\mathbb{F})$, for any field $\mathbb{F}$, acting regularly on the set $\{(1, v) : v \in \mathbb{F}^n\}$ of affine points. We write the elements of such subgroups R (called regular subgroups) as matrices $\left(\begin{smallmatrix} 1 & v \\ 0 & I_n+\delta_R(v) \end{smallmatrix}\right)$, where $\delta_R : \mathbb{F}^n \to \mathrm{Mat}_n(\mathbb{F})$.

We are interested in classifying regular subgroups R such that the function δ_R is $\mathbb{F}$-linear. Observe that if R is abelian, then δ_R is linear and that if δ_R is linear, then R is unipotent (but not necessarily abelian). Also, δ_R is linear if and only if $\mathrm{A} = \mathbb{F}\, I_{n+1} + R$ is a split local subalgebra of $\mathrm{Mat}_{n+1}(\mathbb{F})$. As firstly observed in [1], two regular subgroups R_1 and R_2, with δ_{R_i} linear, are conjugate in $\mathrm{AGL}_n(\mathbb{F})$ if and only if the corresponding local algebras A_i are isomorphic.

Our methods, based on classical results of linear algebra, allow to classify, up to conjugation, certain types of regular subgroups of $\mathrm{AGL}_n(\mathbb{F})$, for $n \leq 4$, and the corresponding algebras, providing an alternative proof of the results of [2], [3] and [5].

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* Joint work with Chiara Tamburini.

THERE ARE STILL MANY GOOD UNSOLVED PROBLEMS IN FINITE ABELIAN GROUPS: THE POINT OF VIEW OF ADDITIVE COMBINATORICS

Alain Plagne
École Polytechnique
91128 Palaiseau Cedex, France
E-mail: plagne@math.polytechnique.fr

In this talk, we shall survey several problems arising in additive combinatorics. For some of them, even the case of abelian groups is unsolved ...

THE ZASSENHAUS p-FILTRATION AND $\mathbb{F}_p$-COHOMOLOGY FOR PROFINITE GROUPS

Claudio Quadrelli *
Department of Mathematics
Ben-Gurion University of the Negev, Israel
E-mail: quadrell@math.bgu.ac.il

For a group G and a prime number p the Zassenhaus p-filtration is the descending series of (characteristic) subgroups $G_{(n)}$, $n \geq 1$, where $G_{(n)}$ is the kernel of the natural morphism of G into the unit group of the quotient $\Bbbk[G]/I^n$, where $\Bbbk$ is a field of characteristic p and I is the augmentation ideal of $\Bbbk[G]$. For a finitely generated pro-p group G the Zassenhaus p-filtration and the cohomology $\mathbb{F}_p$-algebra $H^\bullet(G,\mathbb{F}_p) = \bigoplus_{n\geq 0} H^n(G,\mathbb{F}_p)$, endowed with the (skew-commutative) cup product, are related by the following (cf. [1]).

Theorem. *Let $G = F/R$ be a minimal presentation, with F a free pro-p group and R generated by $r_1,\ldots,r_m$ as normal subgroup of F.*

1. *The cup product $H^1(G,\mathbb{F}_p)^{\otimes 2} \to H^2(G,\mathbb{F}_p)$ is surjective if, and only if, $r_i \in F_{(2)} \smallsetminus F_{(3)}$ for all i.*
2. *If $H^\bullet(G,\mathbb{F}_p)$ is quadratic then $H^\bullet(G,\mathbb{F}_p)^! = \frac{\mathbb{F}_p\langle X\rangle}{\langle \bar{r}_1,\ldots,\bar{r}_m\rangle}$, with $\mathbb{F}_p\langle X\rangle$ the free associative algebra on $X = \{X_1,\ldots,X_{\mathrm{rk}(G)}\}$ and $\bar{r}_i$ the image of r_i in $F_{(2)}/F_{(3)}$ via the magnus morphism. Moreover, the quadratic dual $H^\bullet(G,\mathbb{F}_p)^!$ is the quadratic cover of the graded $\mathbb{F}_p$-algebra $\bigoplus_{n\geq 0} I^n/I^{n+1}$ induced by $\mathbb{F}_p[G]$. In particular $\bigoplus I^n/I^{n+1}$ is quadratic if, and only if, it is isomorphic to $H^\bullet(G,\mathbb{F}_p)^!$.*

A graded algebra is quadratic if it is generated by elements of degree 1 and its defining relations are generated by homogeneous relations of degree 2, and the quadratic dual $H^\bullet(G,\mathbb{F}_p)^!$ is the algebra generated by the dual of $H^1(G,\mathbb{F}_p)$ and whose defining relations are orthogonal to the defining relations of $H^\bullet(G,\mathbb{F}_p)$. Interesting examples of pro-p groups satisfying statement 2. (and with relevant Galois-theoretic implications) are provided by one-relator pro-p groups satisfying 1. (cf. [2]).

* Joint work with J. Mináč, F. Pasini and N.D. Tân.

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ON THE LENGTH OF A FINITE GROUP

Pavel Shumyatsky
Department of Mathematics
University of Brasilia
Brasilia, Brazil
E-mail: pavel2040@gmail.com

Every finite group G has a normal series each of whose factors either is soluble or is a direct product of nonabelian simple groups. In recent joint work with E. Khukhro we defined the nonsoluble length $\lambda(G)$ as the number of nonsoluble factors in a shortest series of this kind. Upper bounds for $\lambda(G)$ appear in the study of various problems on finite, residually finite, and profinite groups. In particular, such bounds played important role in the Hall-Higman reduction theorem for the restricted Burnside problem. In the talk several new results on $\lambda(G)$ will be discussed.

GENERALIZED HULTMAN NUMBERS, AND NEW GENERALIZED HULTMAN NUMBERS, AND THEIR CONNECTION TO GENERALIZED COMMUTING PROBABILITY

Robert Shwartz [*]
Department of Mathematics
Ariel University, Israel
E-mail: robertsh@ariel.ac.il

Let G be a finite group, and let π be a permutation from S_n. We study the distribution of probabilities of equality

$$a_1 a_2 \cdots a_{n-1} a_n = a_{\pi_1}^{\epsilon_1} a_{\pi_2}^{\epsilon_2} \cdots a_{\pi_{n-1}}^{\epsilon_{n-1}} a_{\pi_n}^{\epsilon_n},$$

when π varies over all the permutations in S_n, and ϵ_i either varies over the set $\{1, -1\}$, or varies over the set $\{1, b\}$, where b is an involution in G (Two different cases of equations). The equation can also be written as

$$a_1 a_2 \cdots a_{n-1} a_n = a_{\pi_1} a_{\pi_2} \cdots a_{\pi_{n-1}} a_{\pi_n},$$

[*] Joint work with Avraham Goldstein, Vadim E. Levit.

where π is a signed permutation from B_n, and each a_{-i} is defined as a_i^{-1} (taking inverse), in case $\epsilon_i \in \{1, -1\}$ or a_{-i} is defined as ba_ib (conjugating by a given involution b), in case $\epsilon_i \in \{1, b\}$. The probability in the second case, depends on the conjugacy class of the involution b. The case, when all ϵ_i are 1, were considered in [1] and led to the Hultman numbers [2]. In this talk we generalize these results in two different ways. In both of the cases we describe the spectrum of the probabilities of signed permutation equalities in a finite group G as the signed permutation varies over all the elements of B_n. This spectrum turns-out to be closely related to the partition of $2^n \cdot n!$ into a sum of the corresponding Generalized Hultman numbers [3] in case of $\epsilon_i \in \{1, -1\}$, or to New Generalized Hultman Numbers, which are introduced in this talk, in case of $\epsilon_i \in \{1, b\}$.

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ON THE HIRSCH-PLOTKIN RADICAL OF STABILITY GROUPS

Gunnar Traustason
University of Bath, U.K.
E-mail: gt223@bath.ac.uk

We will discuss the stability group of subspace series of infinite dimensional vector spaces. C. Casolo and O. Puglisi have shown that when the vector space has countable dimension then the Hirch-Plotkin radical of the stability group coincides with the set of all space autmorphisms that fix a finite subseries and they conjectured that this would hold in all dimensions. We present a counter example of dimension $2^{\aleph_0}$ and discuss some related results.

DECOMPOSITION OF UNIPOTENTS: COMING OF AGE

Nikolai Vavilov
Department of Mathematics and Mechanics
Saint Petersburg State University, Russia
E-mail: nikolai-vavilov@yandex.ru

The talk, which is a continuation of my talk at Ischia 2014, is devoted to the latest progress on and the new applications of a powerful geometric/representation theoretic approach towards the structure theory of algebraic groups over commutative rings, decomposition of unipotents.

Let Φ be a root system, R be a commutative ring, and let $G(\Phi, R)$ be a Chevalley group of type Φ over R. Further, denote by $x_\alpha(\xi)$, where $\alpha \in \Phi$ and $\xi \in R$, the corresponding elementary root element.

For the group $\mathrm{SL}(n, R)$ this method was initially proposed in 1987 by Alexei Stepanov, to give a simplified proof of Suslin's normality theorem. Soon thereafter I generalised it to split classical groups, and then together with Eugene Plotkin we generalised it to exceptional Chevalley groups.

In the simplest form, at the level of K_1, this method gives explicit polynomial factorisations of root type elements $gx_\alpha(\xi)g^{-1}$, where $g \in G(\Phi, R)$, in terms of factors sitting in proper parabolic subgroups, and eventually in terms of elementary generators. Among other things, this allows to give extremely short and straightforward proofs of the main structure theorems for such groups.

However, for exceptional groups the early versions of the method relied on the presence of huge classical embeddings, such as $\mathrm{A}_5 \leq \mathrm{E}_6$, $\mathrm{A}_7 \leq \mathrm{E}_7$ and $\mathrm{D}_8 \leq \mathrm{E}_8$. Also, even for some classical groups, the method would not give explicit bounds on the length of elementary decompositions of root elements.

Inspired by the A_2-proof of structure theorems for Chevalley groups, proposed by myself, Mikhail Gavrilovich, Sergei Nikolenko, and Alexander Luzgarev, recently we succeeded in developing new powerful versions of the method, that only depend on the presence of small rank classical embeddings. In 2014–2015 the following amazing progress occured.

• Combining decomposition of unipotents with localisation and amalgam methods, Andrei Lavrenov and Sergei Sinchuk succeded in obtaining generalisations at the level of K_2, which led to a solution of a 40++ years old open problem, centrality of K_2 modeled on $\mathrm{Sp}(2l, R)$, $\mathrm{Spin}(2l, R)$, $l \geq 3$, and the Chevalley groups of types E_6, E_7 and E_8.

• For the microweight case, Victor Petrov discovered a completely new approach towards the construction of small unipotents stabilising a vector from the highest weight orbit. Unlike original proofs, his construction does not rely on difficult explicit calculations, and uses only some elementary algebro-geometric facts such as motivic filtrations.

• Using generic constructions, Alexei Stepanov gave a new proof of the normal structure theorem for Chevalley groups, that does not invoke the full force of the method, and only relies on the presence of one non-trivial unipotent in the stabiliser of a column.

Combined, these advances made the whole machinery much more manageable and flexible, and paved way for new applications. For instance, it turned out that decompositions of unipotents do not have to be parabolic, they can work in terms of proper reductive subgroups, which allows to dramatically reduce length bounds. I plan to sketch some typical recent applications using this idea.

Finally, I plan to mention possible generalisations to isotropic (but non necessarily split) reductive groups. In this generality, Victor Petrov and Anastasia Stavrova initiated the use of localisation methods, further developed by Stavrova, Luzgarev, Stepanov, and others. Decomposition of unipotents would constitute a viable alternative to and in some cases enhancement of localisation methods, and might have serious impact on the structure theory of such groups. Actually, in many important cases it gives much better length bounds in terms of elementary generators. Another tantalising challenge would be to extend these methods to infinite-dimensional algebraic-like groups such as Kac—Moody groups, etc.

FIRST-ORDER GROUP THEORY AND BRANCH GROUPS

John Wilson
Mathematical Institute, University of Oxford
Oxford, U.K.
E-mail: wilsonjs@maths.ox.ac.uk

The lecture will begin with brief introductions to first-order group theory and to branch groups. Then the first-order interpretability within the groups of the actions of branch groups on the ambient trees will be discussed.

AUTOMORPHISMS OF THE RATIONAL GROUP ALGEBRAS OF FINITE GROUPS

Alexandre E. Zalesski*
Department of Physics, Mathematics and Informatics
National Academy of Science, Belarus
E-mail: alexandre.zalesski@gmail.com

This work is inspired by the following result by W. Feit and G. Seitz [1].

Theorem. *The rational group algebra of a finite group $G \neq \{1\}$ has an outer automorphism.*

We generalize this by proving

Theorem. *The rational group algebra of a finite group $G \neq \{1\}$ has an outer automorphism of order 2, unless G is isomorphic to $SL_2(8)$ or ${}^3D_4(2)$.*

In other words, there are only three groups listed above with the group of outer automorphisms of their group algebra over the rationals is of odd order.

The proof uses the classification of finite simple groups (likewise the proof of Feit-Seitz's theorem does). The proof naturally splits to two parts. First, using character theory and some observations of Feit and Seitz we prove the result for simple groups. The second part is a reduction of the problem to simple groups. Both the cases require non-trivial analysis.

A similar result remains true for group algebras over arbitrary ground field of characteristic 0 or coprime to $|G|$. The cases of modular group algebras or group ring over integers are much more complex and no conjectures are available to the date.

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* Joint work with M. Dokuchaev (Univ. of Saõ Paolo, Brazil).

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REPRESENTATION ZETA FUNCTIONS OF ARITHMETIC GROUPS

Michele Zordan
Department of Mathematics
University of Bielefeld, Germany
E-mail: michele.zordan@math.uni-bielefeld.de

We introduce new methods from representation theory of algebraic groups into the study of representation zeta functions associated with compact $\mathfrak{p}$-adic analytic groups and arithmetic groups. We apply these new methods to compute the representation zeta functions of principal congruence subgroups of the groups $\mathrm{SL}_4(\mathfrak{o})$, where $\mathfrak{o}$ is a compact discrete valuation ring of characteristic 0.

Theorem. *Let $\mathfrak{o}$ be a compact discrete valuation ring of characteristic 0 whose residue field has cardinality q and characteristic not equal to 2. Then, for all permissible m,*

$$\zeta_{\mathrm{SL}_4^m(\mathfrak{o})} = q^{15m}\frac{\mathcal{F}(q,q^{-s})}{\mathcal{G}(q,q^{-s})}$$

where

$$\begin{aligned}
\mathcal{F}(q,t) = qt^{18}& \\
&- \left(q^7+q^6+q^5+q^4-q^3-q^2-q\right)t^{15} \\
&+ \left(q^8-2\,q^5-q^3+q^2\right)t^{14} \\
&+ \left(q^9+2\,q^8+2\,q^7-2\,q^5-4\,q^4-2\,q^3-q^2+2\,q+1\right)t^{13} \\
&- \left(q^{10}+q^9+q^8-2\,q^7-2\,q^6-2\,q^5+2\,q^3+q^2+q\right)t^{12} \\
&+ \left(q^8+2\,q^6+q^4-q^3-q^2-q\right)t^{11} \\
&+ \left(q^8+q^7-2\,q^4+q\right)t^{10} \\
&- \left(2\,q^{10}+q^9+q^8-q^7-3\,q^6-2\,q^5-3\,q^4-q^3+q^2+q+2\right)t^9 \\
&+ \left(q^9-2\,q^6+q^3+q^2\right)t^8 \\
&- \left(q^9+q^8+q^7-q^6-2\,q^4-q^2\right)t^7 \\
&- \left(q^9+q^8+2\,q^7-2\,q^5-2\,q^4-2\,q^3+q^2+q+1\right)t^6 \\
&+ \left(q^{10}+2\,q^9-q^8-2\,q^7-4\,q^6-2\,q^5+2\,q^3+2\,q^2+q\right)t^5 \\
&+ \left(q^8-q^7-2\,q^5+q^2\right)t^4 \\
&+ \left(q^9+q^8+q^7-q^6-q^5-q^4-q^3\right)t^3 \\
&+ q^9 \\
\mathcal{G}(q,t) =& q^9\left(1-qt^3\right)\left(1-qt^4\right)\left(1-q^2t^5\right)\left(1-q^3t^6\right).
\end{aligned}$$