ISCHIA GROUP THEORY 2010

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Periodic groups saturated by dihedral subgroups

Bernhard Amberg

It is well-known that two distinct involutions in any group generate a dihedral group. A group G is called saturated by dihedral groups if every finite subgroup is contained in a finite dihedral group. It is easy to see that every locally finite group G which is saturated by dihedral subgroups is a locally finite dihedral group, i.e. it is a union of an infinite ascending chain of finite dihedral subgroups. Shlyopkin and Rubashkin (2005) have shown that this also holds for some classes of periodic groups, for instance when the elements of G have bounded period or when each pair of conjugate elements of prime order generate a finite group. In other cases they show that G has a factorization as $G = ABC = ACB = BCA = CBA$ where A is (locally) finite dihedral and B, C are (locally) cyclic subgroups.

In some joint work with L.Kazarin we show that in fact every periodic groups saturated by dihedral groups is locally finite dihedral. Some other recent results involving the existence of involutions are also discussed.

A survey on spherical designs, with emphasis on the aspects related to finite groups

Eiichi Bannai

Our aim is to study good finite subsets (finitely many points) of the sphere. First we review the theory of spherical codes and designs following Delsarte-Goethals-Seidel (1977). Here, the theory of association schemes plays an interesting role. There are natural lower bounds for the size of spherical t-designs, and those which attain one of such lower bounds are called tight spherical t-designs. We discuss the known examples of tight spherical t-designs, and survey the current status of the classification of tight spherical t-designs. Moreover, we will discuss spherical designs which are obtained as orbits of a finite subgroup of the real orthogonal group, or spherical designs which are obtained as shells of a Euclidean lattice. If time permits, I will also discuss Euclidean t-designs (a generalization of spherical t-designs), and how the theory of coherent configurations plays a similar role as the theory of association schemes in spherical designs.

Normal coverings of the symmetric and alternating group

Daniela Bubboloni

In this talk we investigate the minimum number of maximal subgroups $H_i,\ i = 1 \dots k$ of the symmetric group S_n (of the alternating group A_n) such that each element in the group S_n (A_n) belongs, up to conjugacy, to some H_i. We show that this number depends on the arithmetical complexity of n and give the exact value when n decomposes in at most two primes, lower and upper bound as well as asymptotic estimate in the general case.

Semi-rational groups

David Chillag
(joint work with S. Dolfi)

A theorem of Gow states that the only prime numbers that can divide the order of a solvable rational group are 2, 3 and 5. We introduce a generalization of rational groups, called semi-rational groups, and show that Gow's like theorem holds.

Subgroups with non-trivial Möbius number in the alternating and symmetric groups

Valentina Colombo

Avinoam Mann has conjectured that if G is a PFG group, then $|\mu(H,G)|$, for any open subgroup H of G, and the number $b_n(G)$ of subgroups of G of index n and non-trivial Möbius number can be bounded polynomially respectively in terms of $|G:H|$ and n. Recently Andrea Lucchini has proved that this problem can be reduced to the study of the almost simple groups; in fact he proved that Mann's conjecture is true if there exist two constant c_1 and c_2 such that for each finite almost simple group X we have: $b_n(X) \leq n^{c_2}$ for each $n \in \mathbb{N}$, and $|\mu(Y,X)| \leq |X:Y|^{c_1}$ for each $Y \leq X$. We have proved the existence of these two constants for all the Alternating and Symmetric groups. We explain the fundamental step in the proof of this result.

Groups with many abelian subgroups

Maria De Falco

In this talk a class of groups with many abelian subgroups will be considered. In particular it will be given a description of groups in which every non-abelian subgroups has finite index.

Groups with finiteness conditions on conjugates and commutators

Francesco de Giovanni

The theory of FC-groups (i.e. of groups with finite conjugacy classes of elements) has been extensively investigated by several authors over the last seventy years. The aim of this talk is to discuss some recent developments of this subject and other related topics.

The Markov-Zariski topology of an infinite group

Dikran Dikranjan

According to Markov, a subset of a group G of the form

$$\{x \in G : x^{n_1}a_1x^{n_2}a_2\dots a_{m-1}x^{n_m} = a_m\},$$

for some natural m, integers $n_1,\dots,n_m$ and elements $a_1,a_2,\dots,a_m \in G$, is an *elementary algebraic set*. The topology $\mathfrak{Z}_G$ on G, obtained by taking as closed sets all intersections of finite unions of elementary algebraic sets was introduced by Bryant under the name *verbal topology*, but it is also widely known under the name *Zariski topology* of G. This is not a group topology. Ths first countably infinite groups with discrete Zariski topology were built by Ol′shankij in 1980 as appropiate quotients of Adian's groups $A(m,n)$.

The $\mathfrak{Z}_G$-closed sets are closed in every Hausdorff group topology on G, i.e., they are *unconditionally closed*, following Markov's terminology. Therefore, the unconditionally closed sets, give rise to a finer topology $\mathfrak{M}_G$, the *Markov topology* of G.

We discuss the properties of these topologies and give a complete description of the Zariski topology of an abelian groups. As an application, we provide partial answers towards two long-standing problems of Markov:

(i) proving the equality $\mathfrak{Z}_G = \mathfrak{M}_G$ for every abelian group;

(ii) providing a characterization of the subsets of an abelian group G that are $\mathcal{T}$-dense in *some* Hausdorff group topology $\mathcal{T}$ on G.

A different proof of (i) was given also by Bryant.

Some stability operations in group theory

Martin Evans

Let F_n denote the free group of rank n and $d(G)$ the minimal number of generators of the finitely generated group G. Given a presentation

$$(1) \quad R \hookrightarrow F_m \overset{\alpha}{\twoheadrightarrow} G$$

of G on $m \geq d(G)$ generators we may view the abelian group $\bar{R} := R/R'$ as a (right) $\mathbb{Z}G$-module on setting $(rR').g = r^{\alpha^{-1}(g)}R'$ where $\alpha^{-1}(g) \in F_m$ is any preimage of $g \in G$ under α and $r^{\alpha^{-1}(g)} = (\alpha^{-1}(g))^{-1}r\alpha^{-1}(g)$, the conjugate of r by $\alpha^{-1}(g)$. We call $\bar{R}$ the *relation module* of G associated with (1) and from now on write it additively.

Suppose that

$$(2) \quad S \hookrightarrow F_m \overset{\beta}{\twoheadrightarrow} G$$

is another m-generator presentation of G. In general $\bar{R} \ncong \bar{S}$ as $\mathbb{Z}G$-modules. On the other hand it is well known that $\bar{R} \oplus (\mathbb{Z}G)^{d(G)} \cong \bar{S} \oplus (\mathbb{Z}G)^{d(G)}$. Thus on applying the 'stability operation' $M \rightsquigarrow M \oplus \mathbb{Z}G$ to both $\bar{R}$ and $\bar{S}$ sufficiently many times we obtain isomorphic modules. We investigate the number of times we need to apply this operation to obtain isomorphic modules.

Similarly, two generating G-vectors $\mathbf{u}, \mathbf{v}$ of length n need not be Nielsen equivalent although on applying the stability operation

$$(a_1, \ldots, a_n) \rightsquigarrow (a_1, \ldots, a_n, 1)$$

sufficiently many times to both $\mathbf{u}$ and $\mathbf{v}$ yields Nielsen equivalent vectors. We investigate the number of times we need to apply this operation to obtain Nielsen equivalent vectors.

Uniform conciseness of outer commutator words

Gustavo A. Fernandez-Alcober

In the 1970's, Jeremy Wilson proved that every outer commutator word ω is concise, i.e. that if ω takes finitely many values in a group G, then the verbal subgroup $\omega(G)$ is finite. In this talk, I present a joint work with Marta Morigi, in which we prove that outer commutator words are furthermore uniformly concise: if an outer commutator word ω takes m values in a group G, then the order of the verbal subgroup $\omega(G)$ is bounded by a function depending only on m, and not on ω or G. For the proof of this result, we introduce a new representation of outer commutator words by means of binary trees.

Finite p-groups with a maximal elementary abelian subgroup of rank 2

George Glauberman
(joint work with N. Mazza)

Suppose p is an odd prime and S is a finite p-group containing a maximal elementary abelian subgroup of order p^2. We show that every elementary abelian subgroup of S has order at most p^p, and we discuss related open questions and applications to simple groups and representation theory.

On infinite Camina groups

Marcel Herzog

A group G is called a Camina group if G' is not equal to G and for each element x of $G \setminus G'$ the conjugacy class of x in G is equal to $xG' = \{xg' | g' \in G'\}$.

The class of finite Camina groups was introduced by Alan Camina in 1978 and since then it had been studied by many authors: Macdonald, Chillag, Mann, Scoppola, Isaacs and Dark. In this joint paper with Patrizia Longobardi and Mercedes Maj, we started the study of infinite Camina groups. We proved, for example, that if G is a locally finite Camina group, then either G/G' is a p-group for some prime p, or G' is a nilpotent group, and we investigated the structure of residually finite Camina groups. On the other hand, we proved that a finitely generated solvable nonabelian Camina group must be finite.

Twisted conjugacy in certain Artin groups and Coxeter groups

Arye Juhasz

As well known, groups are defined by a binary, a unary and a nullary operations (multiplication, inverse and identity) and the relations among them. However, in the course of development of the theory three other operations, derived from these, were introduced. Namely, raising an element to its nth power for some fixed natural number n, conjugating an element a by an element b: $T_b(a) = bab^{-1}$ and commutation which sends (a, b) to $aba^{-1}b^{-1}$. These operations and the relationship among them are used to identify specific classes of groups, like abelian groups, nilpotent groups, solvable groups, etc. The conjugacy operation defines an equivalence relation on the elements of the group and the equivalence classes - the conjugacy classes - are fundamental ingredients of the theory of finite groups and in particular of their representation theory. In the case of infinite groups, it is a problem of fundamental importance to decide whether two given elements of the group belong to the same equivalence class, i.e., are conjugate. This problem is known as the *Conjugacy Problem.* This is a very important problem which is known to be unsolvable in general, but known to be solvable in large classes of groups.

In certain problems in group theory (finite and infinite) and also outside group theory, like in Topology, Fixed-Point Theory, Dynamics and Geometry, the given group is coupled with a specific automorphism F. In this situation one is often interested in the conjugacy operation twisted by F which sends a to $baF(b)^{-1}$. (For example, let k be a finite fild and let K be a finite extension of k with Frobenius automorphism σ of K with respect to k. Then σ acts naturally on $GL_n(K)$ as an automorphism with fixed point set $GL_n(k)$. In this case the subject of study is the pair $(GL_n(K), \sigma)$ rather than $GL_n(K)$ alone.) This operation again induces an equivalence relation on the group and it is of fundamental importance to decide whether two elements of the group are equivalent. This is the *Twisted Conjugacy Problem.* It has been solved for special classes of groups.

In this talk we solve algorithmically this problem in a class of Artin and Coxeter groups. Our main result includes the following.

Theorem. Let G be an Artin group or a Coxeter group defined by a connected Coxeter graph Γ on n vertices, $n \geq 3$, such that if we follow the convention that every pair of vertices (v_i, v_j) for which there is a defining relation of length $2m_{ij}$ among the standard defining relations of G, we introduce an edge labelled with $m_{i,j}$, then Γ contains no triangles, and every m_{ij} is at least 3. Then:

(a) G has solvable twisted conjugacy problem with respect to every automorphism σ of G.

(b) If H is a σ-invariant parabolic subgroup, h_1 and h_2 elements of H, then they are σ-conjugate in G if and only if they are σ-conjugate in H.

We also consider the embedding of the corresponding Artin monoid in G from the point of view of σ-conjugacy.

Group-theoretic applications of Lie rings with finite cyclic grading

Evgenii Khukhro

Applications in group theory of various results on graded Lie rings are discussed. Graded Lie rings appear naturally in the study of automorphisms or derivations, with grading components defined by eigenspace decomposition. for example, the well-known theorems on solubility and nilpotency of Lie rings with a fixed-point-free automorphism of finite order n are essentially about $(\mathbb{Z}/n\mathbb{Z})$-graded Lie rings $L = \bigoplus_{i=0}^{n-1} L_i$ with trivial zero-component $L_0 = 0$. These theorems have been generalized in several directions: for the case of bounded, rather than trivial, L_0, for the case of few non-trivial components, etc. All these results find applications in group theory.

Lifts and generalized vertices for Brauer characters of solvable groups

Mark Lewis
(joint work with J.P. Cossey)

Let G be a solvable group and let ϕ be a p-Brauer character of G where p is an odd prime. We say χ is a lift of ϕ if ϕ is the restriction of χ to the p-regular elements of G. We show that the generalized vertices for χ are all conjugate and if (Q, δ) is a generalized vertex for χ, then Q is a vertex for ϕ and δ is linear. With this result in hand, we show that if ϕ has an abelian vertex Q, then ϕ has at most $|Q : Q'|$ lifts. Finally, we discuss what is needed to prove this for any vertex Q.

On profinite groups with polynomially bounded Möbius numbers

Andrea Lucchini

Let G be a finitely generated profinite groups. We may define the Möbius function $\mu(H, G)$ in the lattice of the open subgroups of G by the rules: $\mu(G, G) = 1$ and $\sum_{K \geq H} \mu(K, G) = 0$ if $H < G$. We will say that a profinite group G has *polynomially bounded Möbius numbers* (PBMN) if $|\mu(H, G)|$ is bounded by a polynomial function in the index of H and the number $b_n(G)$ of subgroups H of index n satisfying $\mu(H, G) \neq 0$ grows at most polynomially in n.

The interest for this question is related to the study of the function $P(G, k)$ expressing the probability that k randomly chosen elements generate G topologically. Indeed the groups G with PBMN are precisely those for which the infinite sum

$$\sum_{H <_o G} \frac{\mu(H, G)}{|G : H|^s}$$

is absolutely convergent in some half complex plane. When this happens, this infinite sum represents in the domain of convergency an analytic function which assumes precisely the value $P(G, k)$ on any positive integer k large enough.

Since $\mu(M, G) = -1$ for any maximal subgroup M of G, it must be $m_n(G) \leq b_n(G)$ (where $m_n(G)$ denotes the number of maximal subgroups of G with index n). In particular, if $b_n(G)$ grows polynomially, then G has polynomial maximal subgroup growth (PMSG). A theorem by Mann and Shalev characterizes groups with PMSG as those which are positively finitely generated (PFG), i.e. $P(G, k) > 0$ for some choice of k. Mann conjectured that, conversely, the following holds:

Conjecture 1. If G is a PFG group, then G has PBMN.

The conjecture has been proved for particular classes of profinite groups, for example some arithmetic groups, finitely generated prosolvable groups, groups with polynomial subgroup growth. We will present a reduction theorem that allows to deal only with finite almost simple groups. We will say that an almost simple group X is (c_1, c_2)-*bounded* if there exist two constants c_1 and c_2 such that

(1) $b_n^*(X) \leq n^{c_1}$, where $b_n^*(X)$ denotes the number of subgroups K of X with $|X : K| = n$ and $X = K socX$;
(2) $|\mu(K, X)| \leq |X : K|^{c_2}$ for each $K \leq X$ with $X = K socX$.

Denote by $\Lambda(G)$ the set of finite monolithic groups L such that $socL$ is nonabelian and L in an epimorphic image of G. If $L \in \Lambda(G)$, then $socL = S_1 \times \ldots \times S_r$, where the S_i's are isomorphic simple groups. Let X_L be the subgroup of $autS_1$ induced by the conjugation action of $N_G(S_1)$ on S_1. This X_L is a finite almost simple group, uniquely determined by L. Our main result is the following.

Theorem 1. *A PFG group has PBMN if there exists c_1 and c_2 such that X_L is (c_1, c_2)-bounded for each L in $\Lambda(G)$.*

This theorem allows us to reformulate Mann's Conjecture as follows.

Conjecture 2. There exist c_1 and c_2 such that any finite almost simple group is (c_1, c_2) bounded.

Groups and Lie rings with Frobenius groups of automorphisms

Natalia Yu. Makarenko

My talk is an overview of recent results (joint with P. Shumyatsky) on finite groups and Lie rings admitting a Frobenius group of automorphism FH with cyclic kernel F. In particular, we show that if a Frobenius group FH with cyclic kernel F and complement H of order q acts by automorphisms on a Lie algebra L in such a way that $C_L(F) = 0$ and $C_L(H)$ is nilpotent of class c, then L is nilpotent of (c, q)-bounded class. This result implies important consequences for finite groups. For instance, we prove that if GFH is a double Frobenius group with complement H of order q such that $C_G(H)$ is nilpotent of class c, then G is nilpotent of (c, q)-bounded class.

Uniform triples

Gunter Malle
(joint work with R. Guralnick)

Let G be a group, V a nontrivial finite-dimensional irreducible kG-module (k any field). Then there exists $g \in G$ with fixed space of dimension at most $(1/3)\dim V$. This solves a conjecture of Peter Neumann from 1966. The crucial case turns out to be when G is finite non-abelian simple. For those groups, we derive a much stronger statement from a particular form of generation by three elements which in turn is proved using character theory.

Uniform growth of free products and related constructions

Avinoam Mann

Let G be a finitely generated group. We write $s(n)$ for the number of elements which can be written as words of length at most n in the generators. The function $s(n)$ grows at most exponentially, and G is said to have *exponential growth* if there exists a number $C > 1$ such that $s(n) > C^n$. This notion is independent of the system of generators. G has *uniform exponential growth*, if C can be chosen independently of the system of generators. It is known that free and amalgamated products, HNN extensions, one relator groups, etc., are usually of uniform exponential growth, with explicitely given values of C. We improve the values of these constants. E.g. for HNN extensions the golden ratio is a universal lower bound.

Average dimension of fixed point spaces

Attila Maroti

Let G be a finite group, F a field, and V a finite dimensional FG-module such that G has no trivial composition factor on V. Then the arithmetic average dimension of the fixed point spaces of elements of G on V is at most $(\frac{1}{p})\dim V$ where p is the smallest prime divisor of the order of G. This answers and generalizes a question of Peter M.Neumann and Michael Vaughan-Lee. Several applications of this result are given. This is a joint work with Robert M. Guralnick.

Pro-p groups with waists

Valerio Monti

A subgroup W of a pro-p group G is said to be a *waist* if for every open normal subgroup N either $N \leq W$ or $N \geq W$. There are several examples of waists: the open subgroups of a procyclic group, the terms of the lower central series of a pro-p group of maximal class, the Camina kernels (i.e. normal subgroups whose nontrivial cosets consist of conjugate elements.

Our main goal is to study the position of a waist W with respect to the terms of a central series of the group G. If p is odd we show that W is a term of both the lower and upper central series of the group G (with a few natural exceptions).

Unramified Brauer groups of finite and infinite groups

Primoz Moravec

The Bogomolov multiplier is a group theoretical invariant isomorphic to the unramified Brauer group of a given quotient space, and represents an obstruction to the problem of stable rationality. We describe a homological version of the Bogomolov multiplier, find a five term exact sequence corresponding to this invariant, and describe the role of the Bogomolov multiplier in the theory of central extensions. We define the Bogomolov multiplier within K-theory and show that proving its triviality is equivalent to solving a long-standing problem posed by Bass. An algorithm for computing the Bogomolov multiplier is presented.

Advances on Brauer's height zero conjecture

Gabriel Navarro

With P.H. Tiep, we prove Brauer's Height Zero Conjecture for the 2-blocks of maximal defect.

Localizations of finitely generated soluble groups

Niamh O'Sullivan

The study of localizations of groups has concentrated on group theoretic properties which are preserved by localization. In this talk we look at finitely generated soluble groups and determine when the local groups associated with them are soluble.

Exterior self-quotient modules

Peter P. Palfy
(joint work with S. Glasby and C. Schneider)

Let $G \leq GL(V)$ act on the exterior product $V \wedge V$ by $(u \wedge v)g = (ug) \wedge (vg)$. We say that V is an *exterior self-quotient G-module*, if $V \wedge V$ has a quotient module isomorphic to V. Then we also say that G is an *ESQ-group.* We study the question for which vector spaces V we can find an irreducible group $G \leq GL(V)$ turning V to an exterior self-quotient G-module. The question arose in our investigation of finite p-groups of exponent p^2 (p odd) with a unique nontrivial proper characteristic subgroup. In the small dimensional cases we found that $GL_d(p)$ contains an irreducible ESQ-group as follows: $d = 3$: if $p \neq 2$; $d = 4$: if $p \neq 5$; $d = 5$: if $p^5 \equiv 1 \pmod{11}$; $d = 6$: always; $d = 7$: if $p \neq 2$.

On the irreducibility of the Dirichlet polynomial of a simple group of Lie type

Massimiliano Patassini

Given a finite group G and a normal subgroup N of G, the Dirichlet polynomial of G given G/N is

$$P_{G,N}(s) = \Sigma_{H \leq G, NH=G} \frac{\mu_G(H)}{|G:H|^s}.$$

The polynomial $P_{G,N}(s)$ is an element of the factorial domain

$$\mathcal{R} = \left\{ \Sigma_{m \geq 1} \frac{a_m}{m^s} : a_m \in \mathbb{Z}, |\{m : a_m \neq 0\}| < \infty \right\}$$

which is called the ring of Dirichlet finite series.

In our work, we suppose that G is monolithic primitive group with non-abelian socle $soc(G) \simeq S^n$ for some simple group S of Lie type. Under some assumptions on the Lie rank of S, we prove that $P_{G,soc(G)}(s)$ is irreducible in $\mathcal{R}$. Moreover, we show that the Dirichlet polynomial $P_S(s) = P_{S,S}(s)$ of a simple group S of Lie type is reducible in $\mathcal{R}$ if and only if S is isomorphic to $PSL_2(p)$ where p is a Mersenne prime such that $\log_2(p+1) \equiv 3 \pmod 4$.

Hurwitz generation of the universal covering of Alt(n)

Marco Pellegrini
(joint work with M.C. Tamburini)

A finite group is said to be Hurwitz if it can be generated by two elements of orders 2 and 3, whose product has order 7. M. Conder classified the alternating groups which are Hurwitz. Starting from his results, we prove that the universal covering of an alternating group which is Hurwitz, is still Hurwitz, with precisely 31 exceptions. Furthermore, for each positive answer we exhibit a $(2, 3, 7)$ generating triple of the universal covering, up to a central element.

Galois invariance, trace and subfield subcodes

Andrea Previtali

Given a Galois extension we relate subfield subcodes with trace codes showing that a code is invariant under the Galois group if and only if its restriction coincides with the trace code.

Sylow permutability in locally finite groups

Derek J.S. Robinson

A subgroup H is said to be Sylow-permutable (or S-permutable) in a group G if $HP = PH$ for every Sylow subgroup P of G. If S- permutability is a transitive relation in G, then G is called a PST-group. Finite PST-groups have been studied intensively in recent years, especially in the soluble case, but little is known about locally finite PST-groups. Here we present a result which describes the structure of locally finite groups all of whose finite subgroups are PST-groups.

Arithmetical properties of finite groups

Wujie Shi

Let G be a finite group and $Ch_i(G)$ be one of the following sets:
$Ch_1(G) = |G|$, that is, the order of G.
$Ch_2(G) = \pi_e(G) = \{o(g)|g \in G\}$, that is, the set of element orders of G.
$Ch_3(G) = cs(G) = \{|g^G||g \in G\}$, that is the set of conjugacy class sizes of G.
$Ch_4(G) = cd(G) = \{ch_i(1)|ch_i(1) \in Irr(G)\}$, that is, the set of irreducible character degree of G.

Our main interesting is to study the group structure of G under certain arithmetical conditions of $Ch_i(G)$, $i = 1, 2, 3$ or 4.

Problem A. If $Ch_i(G)$ is fixed, what can we say about the structure of the group G?

For example we have proved that any finite simple group G can be determined by its order $|G|$ ($Ch_1(G)$) and the set $\pi_e(G)$ ($Ch_2(G)$), the set of element orders of G.

$Ch_3(G) = cs(G) = \{|g^G||g \in G\}$ related to J.G. Thompson's conjecture; and $Ch_4(G) = cd(G)$, related to Huppert's conjecture.

Problem B. For the set $Ch_i(G)$, $i = 2, 3, 4$, we can define a graph $\Gamma_i(G)$ as follows: its vertices are the primes dividing the number of $Ch_i(G)$; and two distinct vertices p, q are connected if $pq|m$ holds for some $m \in Ch_i(G)$.

If we know some information of the graph $\Gamma_i(G)$, what can we say about the structure of the group G?

Commutators in residually finite groups

Pavel Shumyatsky

Given a positive integer n, we consider residually finite groups G in which all commutators have order dividing n. In the talk we will discuss some sufficient conditions for G' to be locally finite. In particular we will present the theorem that if every product of 68 commutators in G has order dividing n, then G' is locally finite.

Group identities for symmetric units of group algebras

Ernesto Spinelli

Let FG be the group algebra of a group G over a field F of characteristic different from 2. If G is endowed with an involution $*$, then it can be extended linearly to an involution on FG, also denoted by $*$. An element α in FG is said to be *symmetric* with respect to $*$ if $\alpha^* = \alpha$.

The symmetric units and the symmetric elements have been the subject of a good deal of attention. In particular, it is interesting to know the extent to which the symmetric units determine the structure of the whole unit group of the group ring. Prior to the last couple of years the attention had been largely devoted to the classical involution induced from the map $g \rightarrow g^{-1}$, $g \in G$. Recently, there has been a considerable amount of work on involutions of FG other than the classical one.

In this talk we shall discuss when the symmetric units satisfy a nilpotency identity. In this framework, an important role will be played by the characterization of group algebras whose symmetric units satisfy a group identity ([1]) and that of group algebras whose symmetric elements are Lie nilpotent ([2]). This is a joint work with Sudarshan K. Sehgal and Gregory T. Lee ([3]).

References

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On the adjoint group of radical rings and related questions

Yaroslav Sysak

The set of all elements of an associative ring R, not necessarily with an identity element, forms a monoid with neutral element $0 \in R$ under the circle operation $r \circ s = r + s + rs$ for all r, s in R. The group of all invertible elements of this monoid is called the adjoint group of R and usually denoted by R° which means that R coincides with its Jacobson radical.

The aim of this talk is to present some results and problems about connections between the structure of a radical ring R and that of its adjoint group R°. In particular, the group and Lie properties of radical rings whose adjoint group satisfies finiteness, Engel or generalized soluble conditions will be considered. Both a survey of the development of the theory and some new results are given. Furthermore, an application of radical rings to the study of factorized groups and some generalizations related to them will also be discussed.

On complete resolutions

Olympia Talelli

Gedrich and Gruenberg (1987) and Ikenaga (1984) studied the algebraic invariants silp$\mathbb{Z}G$, the supremum of the injective lengths of the projective $\mathbb{Z}G$- modules, and spli$\mathbb{Z}G$, the supremum of the projective lengths of the injective $\mathbb{Z}G$- modules, in connection with the existence of complete cohomological functors for the group G and showed that these are related to the virtual cohomological dimension of G. Here we show that silp$\mathbb{Z}G$ and spli$\mathbb{Z}G$ are related to the Gorenstein dimension of $\mathbb{Z}G$ and to the existence of a finite dimensional model for E_G, the classifying space for proper actions.

Uniform (2,k)-generation of matrix groups of small rank

M. Chiara Tamburini

It is well known that all finite simple groups can be generated by two elements. This result was proved by Miller (1901) for the alternating groups, Steinberg (1962) for the groups of Lie type and Aschbacher- Guralnick (1984) for the sporadic groups. Actually there is a great abundance of generating pairs: indeed two randomly chosen elements of a finite simple group G generate G with probability $\to 1$ as $|G| \to \infty$, as shown by Kantor-Lubotzky (1990) and Liebeck and Shalev (1995). This fact allows many modifications of the 2-generation problem and predicts that it may be more difficult for small groups. in particular the constructive approach has been considered by many authors, in view of its applications.

Let us recall that a group is said to be (n, k)- generated if it can be generated by a pair of elements of respective orders n and k. The aim of this talk is to illustrate some methods which lead to construct $(2, k)$-generating pairs for classical matrix groups of small rank. Such generators are also uniform, in the sense that the shape of the generators is the same for all groups in a given class.

The results are joint work with M. Vsemirnov and M. Pellegrini.

Symplectic nil-algebras

Maria Tota

Symplectic alternating algebras have arisen in the study of 2-Engel groups ([1], [2]) and they have been studied in more details in [3]. An ongoing joint work with A. Tortora and G. Traustason tries to get an answer to the following question raised by Traustason ([3]):

Question. What can one say about the structure of symplectic nil-algebras, that is symplectic alternating algebras satisfying the extra law $yx^n = 0$ for some positive integer n. In particular, does a symplectic nil-algebra have to be nilpotent?

In this talk, we will see what happens when $n = 2$ and $n = 3$.

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Right Engel subgroups

Gunnar Traustason

We say that a normal subgroup H of a group G is a right n-Engel subgroup if all the elements of H are right n-Engel. The nth term of the upper central series is always a right n-Engel subgroup and allthough it is conversely not true in general that normal right n-Engel subgroups are contained in tha Hypercentre this is true for many classes of groups including all finite groups.

Motivated by some strong structure results for n-Engel groups we will discuss some recent generalizations giving criteria for a right n-Engel subgroup to be contained in the hypercentre.

The Brauer-Clifford group of G-rings

Alexandre Turull

Let G be a finite group. Traditionally, the representation theory of G considers a commutative ring R, and studies modules M over R on which G acts where the actions of R and G on M obey some compatibility properties. When R is instead a commutative G-ring, one defines similarly G-modules over R, where the compatibility now takes into account the action of G on R. The traditional notion of module M over a ring R is then simply a G-module over the G-ring R, where we take the action of G on R to be trivial. The author has defined the notion of the Brauer-Clifford group of certain G-algebras over fields. The Brauer-Clifford group is useful for the study of Clifford theory of finite groups, and in particular, it has been used to prove a strengthened version of the

McKay Conjecture for all finite p-solvable groups. We see how, by incorporating the study of G-modules over commutative G-rings, we may give a natural definition of the Brauer-Clifford group of G-rings. This simpler definition extends the definition of the Brauer-Clifford group, and provides a more flexible basis for applications to the Clifford theory of finite groups.

Diameters of soluble groups, and applications

John S. Wilson

Estimates are given for the diameters of Cayley graphs of soluble subgroups of finite general linear groups, and some applications are discussed.

On the decomposition numbers for projective characters of Chevalley groups

Alexandre Zalesski

Let G be a finite group and p a prime number. Representations of G over an algebraically closed field of characteristic p are called p–modular, and those over the complex numbers are called ordinary. An ordinary representation is equivalent to a representation ϕ over a finite extention of the rationals, and moreover, over a maximal subring R of the latter field not containing p. In addition, R has a unique maximal ideal I such that $F = R/I$ is a finite field of characteristic p. If $\phi(G) \subset GL(n, R)$ for some n, then the representation $\bar{\phi} : G \to GL(n, F)$ obtained from the natural projection $GL(n, R) \to GL(n, F)$ is called the reduction of ϕ modulo p. If ϕ is irreducible then $\bar{\phi}$ is not always irreducible. Let $\bar{F}$ be the algebraic closure of F. The multiplicities of irreducible representations of G over $\bar{F}$ as constituents of $\bar{\phi}$ are called the decomposition numbers.

For applications it is important to know the decomposition matrices for various groups. The theory of modular representations develops methods for understanding properties of the matrix. Brauer's theory of blocks is one of the most powerful in the theory. However, this does not help for groups such as $PSL(d, p)$ as in this case there are only two Brauer blocks. There is another approach, via principal indecomposable $\bar{F}G$–modules, also developed by Brauer. I shall explain some recent results, which demonstrate that this approach is rather efficient and has a lot of potential when G is a Chevalley group in defining characteristic p.