Aoki-Yoshikawa model

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AYM

Economic interpretation	* Physical interpretation
n = total endowment of production	n = # of particles in the system
g = # economic sectors	g = # energy levels
n_i = amount of production of the i-th economic sector	n_i = # particles belonging to the i-th energy level
a_i = level of productivity of the i-th economic sector	ε_i = energy of the i-th level $\varepsilon_i = i \cdot \varepsilon = i \quad \forall i = 1, \dots, g$ $\varepsilon = 1$
D = total (or aggregate) demand	E = total energy of the system

CONSTRAINTS

1) $\sum_{i=1}^{g} n_i = n$ 1) "

2) $\sum_{i=1}^{g} a_i \cdot n_i = D$ 2) $\sum_{i=1}^{g} \varepsilon_i \cdot n_i = E$

DYNAMIC OF AYM

$$\underline{n} = (n_1, .., n_g) \longrightarrow \underline{n}_{ij}^{lm} = (n_1, .., n_i - 1, ..., n_j - 1, ..., n_l + 1, ..., n_m + 1, ..., n_g)$$

Destruction prob: $\underline{P}(\underline{n}_i | \underline{n}) = \frac{n_i}{n}$ $\forall i$

Creation prob: $\underline{P}(\underline{n}^l | \underline{n}) \propto (1 + c \cdot n_l)$ $\forall l$

c = model parameter

- c = 0 (MB case)
- c = 1 (BE case)
- c = -1 (FD case)

Why $d_{max} = E - (n-1)$?

If $n = 5$ particles are in the 3rd energy level, then $E = 5 \cdot 3 = 15$.

→ Is it possible to reach the energy level $\underline{15 - (5-1)} = \underline{11}$?

Yes!

1 particle → 11 th energy level $\quad 1 \cdot 11 + 4 \cdot 1 = 15$ ✓

4 particles → 1st energy level

→ Is it possible to reach the energy level 12 ?

No!

1 particle → energy level 12 $\quad 1 \cdot 12 + 4 \cdot 1 = 16 > 15$ ✗

4 particles → 1st energy level

$n \quad E \quad d_{max} = E - (n-1)$

If $en2 = \text{floor}(en/2)$, then one of the particles reaccommodates in an energy level $j \leq en2$ and the other in the symmetric energy level $en - j$.

proof

1st case $j = \frac{en}{2}$

the other particle needs to be allocated in a level h

$$j + h = en \Rightarrow h = en - j = en - \frac{en}{2} = \frac{en}{2}$$

$\Rightarrow$ the 2 particles reaccommodate in the same energy level $en/2$.

2nd case $j \neq \frac{en}{2}$

N.A. $\left\{ \text{2 particles} \begin{array}{l} \nearrow l < \frac{en}{2} \\ \searrow m < \frac{en}{2} \end{array} \right.$ $E_R = l + m < \frac{en}{2} + \frac{en}{2} = en$ $\times$

N. A. $\left\{ 2 \text{ particles} \begin{cases} l > \frac{ln}{2} \\ m > \frac{ln}{2} \end{cases} \right.$

$$E_R = l + m > \frac{ln}{2} + \frac{ln}{2} = ln$$

A. $\left\{ 2 \text{ particles} \begin{cases} j < \frac{ln}{2} \\ h > \frac{ln}{2} \end{cases} \right.$

$$j + h = ln$$

$$\Rightarrow h = ln - j$$

$$\mathbb{E}(N_i) = \frac{g_i}{e^{\beta\varepsilon_i - \nu} - c} = e^{\nu - \beta i} = \frac{n^2}{E-n} \cdot \left(\frac{E-n}{E}\right)^i$$

$g_i = 1 \quad \forall i$

$c = 0$

$\varepsilon_i = i \quad \forall i$

$\beta = \ln\left(\frac{E}{E-n}\right)$

$\nu = \ln\left(\frac{n^2}{E-n}\right)$

obtained by maximizing the eq. (21) of Scalas5

$$= \left(\frac{n}{\pi - 1}\right) \cdot \left(\frac{\pi - 1}{\pi}\right)^i .$$

$\pi := \frac{E}{n}$ (= level)

Exercise1-Scalas5.R

Source on Save Run Source

```r
# Program binary.R
# This program simulates the Ehrenfest-Brillouin model with binary moves and
# the constraint on energy (or demand)
# The energy (or demand) quantum is e = 1
n<-30 # number of particles
c<-0 # c=1 for BE, c=0 for MB (c=-1 for FD)
T<-10000 # number of Monte Carlo steps
# We are only interested in the transitions
# between energy (or productivity) levels!
# Initial occupation vector. We start with all
# particles (workers) in a given cell of a given level, in this case 3.
# This fixes the energy (or production) of the system
level<-3
E<-level*n # total initial energy (production)
dmax<-E-(n-1) # the maximum level that can be occupied
i<-c(1:dmax) #number of levels
N<-rep(c(0), times=dmax) #level occupation vector
EN<-rep(c(0), times=dmax) # time average of N
probd<-rep(c(0),times=dmax) #vector of hypergeometric probabilities
N[level]<-n #all the particles are in the given level
A<-N #vector of results
# Monte Carlo cycle
for (t in 1:T){
  # first destruction (an object is removed from an
  # occupied category according to a hypergeometric probability)
  probd<-N/n
  cumprobd<-cumsum(probd)
  indexsite1<-min(which((cumprobd-runif(1))>0)) # level 1
  N[indexsite1]<-N[indexsite1]-1
```

Exercise1-Scalas5.R

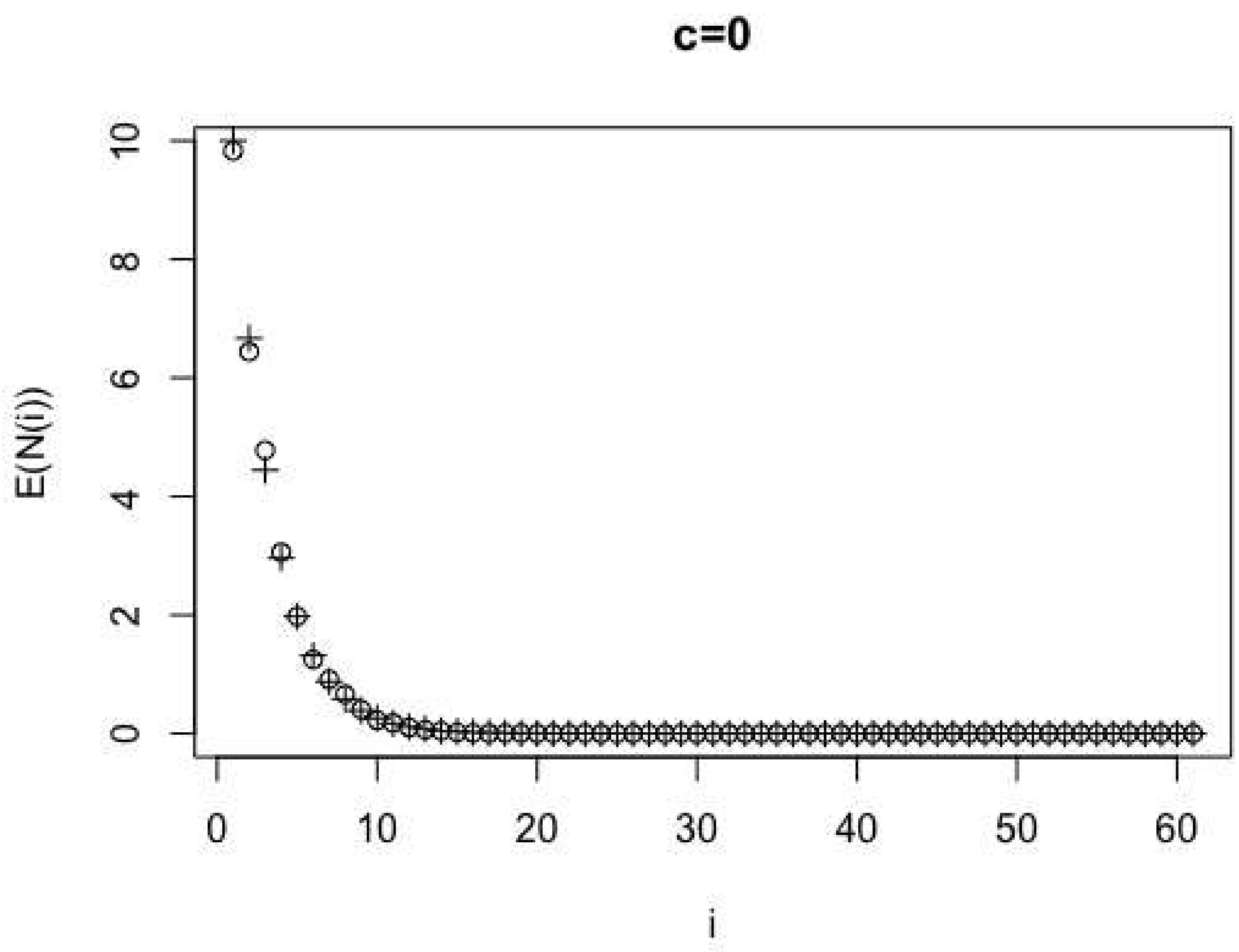
Source on Save Run Source

```r
  # second destruction (an object is removed from
  # an occupied category according to a hypergeometric probability)
  probd<-N/(n-1)
  cumprobd<-cumsum(probd)
  indexsite2<-min(which((cumprobd-runif(1))>0)) #level 2
  N[indexsite2]<-N[indexsite2]-1
  en<-indexsite1+indexsite2 # energy (productivity) of the
  #destroyed (removed) particles (workers)
  en2<-floor(en/2)
  probc<-rep(c(0), times=en2) #probability of creation
  # creation (the two particles reaccomodate
  #according to a Polya probability)
  for(j in 1:en2) {
    probc[j] <- 2*(1+c*N[j])*(1+c*N[en-j]) # I can insert one element in the j-th category
    # and the other in the (en-j)-th category or viceversa (for this reason we have the product times 2)
    if (j==en-j) { # when j=en-j, the selected category is only one, then
      # we have insert the two elements in the j-th category
      probc[j] <- probc[j]/2
    } #end if
  } # end for j
  cumprobc<-cumsum(probc/sum(probc))
  indexsite3<-min(which((cumprobc-runif(1))>0)) #level 3
  N[indexsite3]<-N[indexsite3]+1
  indexsite4<-en-indexsite3 #level 4
  N[indexsite4]<-N[indexsite4]+1
  EN<-EN+N/T # the average of N is stored
  A<-c(A,N) #data update
} #end for on t
```

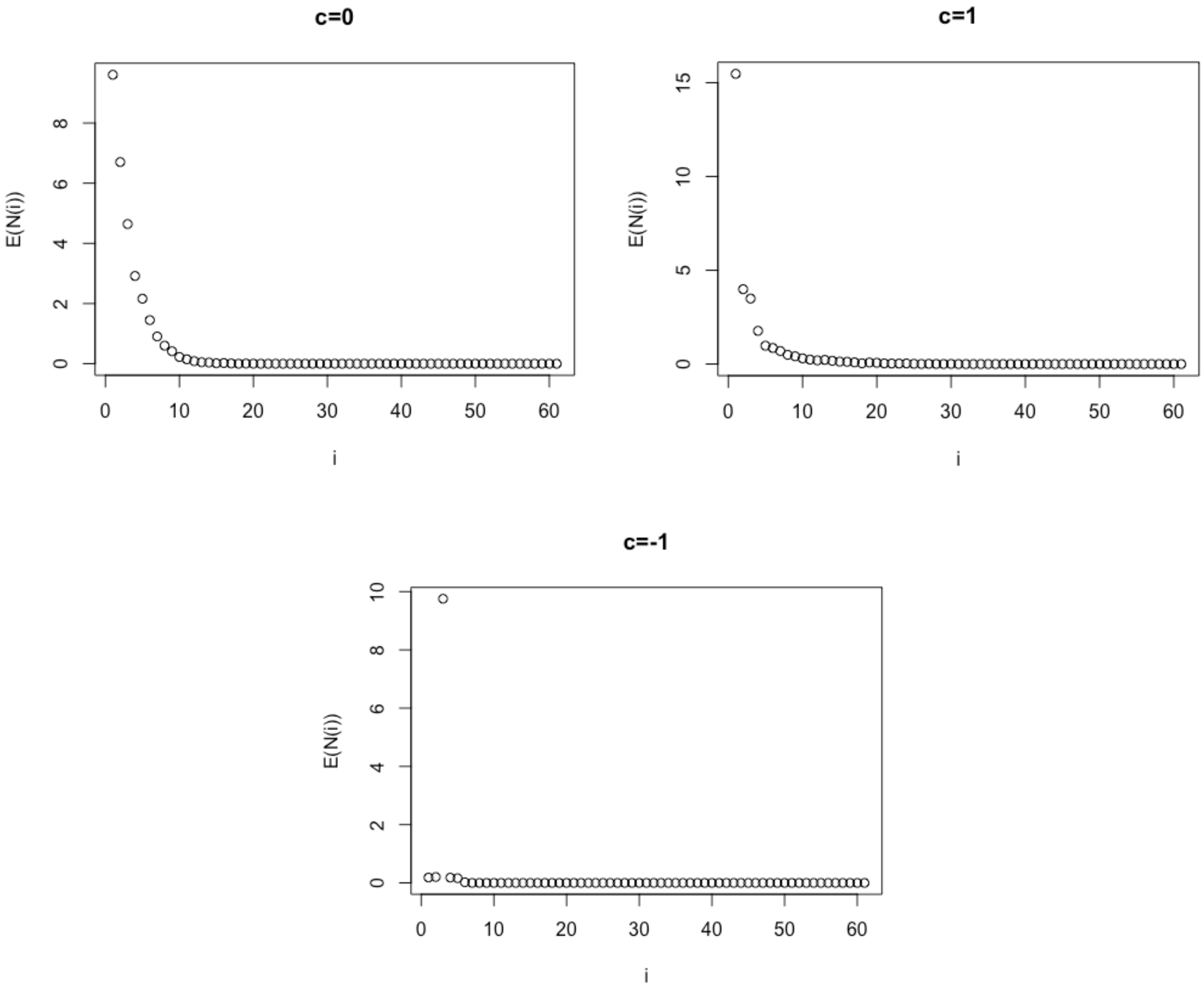
Exercise1-Scalas5.R

Source on Save | Run | Source

```r
A<-matrix(A,nrow=T+1,ncol=dmax,byrow=TRUE)
k<-c(1:dmax)
plot (k, (EN), xlab="i", ylab="E(N(i))",main="c=0")
ENth<-(n*((level-1)/level)^k)/(level-1) #c=0
#ENth <- ((E/(E-n))^k*(E-n)/(n^2)-c)^(-1)
lines(k, (ENth), type="p", pch=3)
```



c=0
E(N(i))
0
2
4
6
8
10
0
10
20
30
40
50
60
i



c=0
c=1
c=-1
E(N(i))
i
0
10
20
30
40
50
60
2
4
6
8
5
15