Exercise 1

venerdì 11 giugno 2021 14:36

Consider a system of 5 "energy elements" (whose value is $\mathcal{E}$) allocated into three molecules.

1) Is it consistent to assume that the partition vector is
$(z_0 = 1,\ z_1 = 0,\ z_2 = 1,\ z_3 = 1,\ z_4 = 0,\ z_5 = 0)$?

YES

$n = 5$ elements

$g = 3$ categories

$$\sum_{i=0}^{3} z_i = 3 \checkmark \qquad \sum_{i=0}^{3} i \cdot z_i = 5 \checkmark$$

$$1 + 0 + 1 + 1 + 0 + 0 = 3$$

$$0 \cdot 1 + 1 \cdot 0 + \underbrace{2 \cdot 1 + 3 \cdot 1}_{=5} + 4 \cdot 0 + 5 \cdot 0$$

2) What is its physical meaning?
$(z_0 = 1,\ z_1 = 0,\ z_2 = 1,\ z_3 = 1,\ z_4 = 0,\ z_5 = 0)$

$z_0 = 1$ = number of categories with 0 element
MOLECULES $\mathcal{E}$

There is one molecule with $0 \cdot \mathcal{E} = 0$ energy

$z_1 = 0$ There are not molecules with 1 energy element

$z_2 = 1$ There is one molecule with $2 \cdot \mathcal{E}$ energy

$z_3 = 1$ There is one molecule with $3 \cdot \mathcal{E}$ energy

$z_4 = 0$ There are not molecules with 4 or 5 energy elem.

$z_5 = 0$

In conel:

one molecule has not energy

one molecule has $2\mathcal{E}$ energy

one molecule has $3\mathcal{E}$ energy

Exercise no.2

venerdì 11 giugno 2021 14:27

Repeat the analysis of the Example on sampling for $n = 3$ and $g = 3$, the following individual description, $\mathbf{X} = (1, 2, 3)$ and a sample of 1 element.

Consider $n = 3$ agents and $g = 3$ strategies (e.g. $1 :=$ "bull" - optimistic, $2 :=$ "bear" - pessimistic, $3 :=$ "neutral" - normal).

The individual description is given by $X = (1, 2, 3)$.

One agent is drawn.

(i) List all the possible sample descriptions $(\mathfrak{Z}_i)_{i=1,\dots,3}$

$$\mathbb{X} = (\underbrace{1, 1, \dots, 1}_{n_1}, \underbrace{2, \dots, 2}_{n_2}, \dots, \underbrace{g \dots g}_{n_g}) \qquad W(\mathfrak{Z}, \underline{n}) = \frac{n!}{n_1!\, n_2! \dots n_g!}$$

$$n = n_1 + n_2 + \dots + n_g$$

$$\mathbb{X} = (1, 2, 3) \qquad W(\mathfrak{Z}, \underline{n}) = \frac{3!}{1!\, 1!\, 1!} = 6$$

$n_1 = 1$ $\quad n_2 = 1$ $\quad n_3 = 1$

$$\underline{n} = (1, 1, 1)$$

$$\{ (1, 2, 3);\ (1, 3, 2);\ (2, 1, 3);\ (2, 3, 1);\ (3, 1, 2);\ (3, 2, 1) \}$$

(ii) Write all possible descriptions of the reduced sample $\mathfrak{Z}_1$

$\{1, 2, 3\}$

$\mathfrak{Z}_1 = 1 \quad \downarrow \quad \underline{m} = (1, 0, 0)$

$\mathfrak{Z}_1 = 2 \quad \downarrow \quad \underline{m} = (0, 1, 0)$

$\mathfrak{Z}_1 = 3 \quad \downarrow \quad \underline{m} = (0, 0, 1)$

(iii) Write all the possible frequency vectors for the reduced sample and count their number.

We have a fixed reduced sequence $\mathfrak{Z}_1$ and we want to know how many individual sequences $\underline{\mathfrak{Z}}$ pass through the given $\underline{m}$

$$\mathfrak{Z}_1 = 1 \longrightarrow \underline{\underline{m} = (1, 0, 0)} \longrightarrow \left.\begin{array}{l} \underline{\mathfrak{Z}} = (1, 2, 3) \\ \underline{\mathfrak{Z}} = (1, 3, 2) \end{array}\right\} \Rightarrow W(\mathfrak{Z} \mid \underline{m}, \underline{n}) = 2$$

$$W(\underline{\xi} \mid \underline{m}, \underline{n}) = \frac{m!}{m_1! \dots m_g!} \cdot \frac{(n-m)!}{(n_1-m_1)! \dots (n_g-m_g)!}$$

$$= \frac{1!}{1!\ 0!\ 0!} \cdot \frac{(3-1)!}{(1-1)!\,(1-0)!\,(1-0)!} = 2$$

$$\xi_1 = 2 \longrightarrow \underline{m} = (0,1,0) \begin{array}{l} \nearrow \underline{\xi} = (2,1,3) \\ \searrow \underline{\xi} = (2,3,1) \end{array} \Bigg\} \Rightarrow W(\underline{\xi} \mid \underline{m}, \underline{n}) = 2$$

$$\xi_1 = 3 \longrightarrow \underline{m} = (0,0,1) \begin{array}{l} \nearrow \underline{\xi} = (3,1,2) \\ \searrow \underline{\xi} = (3,2,1) \end{array} \Bigg\} \Rightarrow W(\underline{\xi} \mid \underline{m}, \underline{n}) = 2$$